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LINEAR POLARIZATION SENSITIVE VLBI

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ABSTRACT We describe the techniques currently used to produce linear polarization sensitive radio images at milliarcsecond resolution, and briefly discuss what will be possible in the next few years.

INTRODUCTION

Measurement of linear polarization at milliarcsecond resolution provides unique information about physical conditions in compact radio sources. One can determine the orientation and order of magnetic fields (through the direction of the electric vectors and the degree of polarization) and see the effects of fluid dynamical structures such as shocks (through their effect on the magnetic field). Polarization observations are sensitive to the bulk motion of the radiating plasma (through relativistic aberration), as well as to the thermal particle environment, both mixed into and surrounding the radiating material (by the Faraday effect). Such measurements are especially valuable at milliarcsecond resolution because they provide this information on the scale of parsecs, enabling study of the physical conditions in the innermost accessible regions of radio jets. In this review we concentrate on the technical aspects of such measurements. A preliminary description of these techniques was given by Roberts *et al.* (1984), and a fuller discussion will be presented by Roberts, Wardle, and Brown (in preparation).

HISTORY OF VERY LONG BASELINE POLARIMETRY

The first successful polarization-sensitive observation at milliarcsecond resolution was that of Cotton *et al.* (1984), who mapped the quasar 3C 454.3 at 2.3 GHz (λ13 cm). Over the last decade we have pursued an extensive program of linear polarization measurements at 5 GHz (λ6 cm), including multi-epoch studies of a large sample of BL Lacertae objects (Roberts, Gabuzda, and Wardle 1987; Gabuzda, Wardle, and Roberts 1989a,b; Gabuzda, Wardle, and Roberts 1989a,b; Gabuzda *et al.* 1989, and in preparation), detailed studies of a small number of quasars (Wardle *et al.* 1986; Roberts *et al.* 1990; Brown *et al.*, in preparation), and a survey of bright northern-hemisphere sources (Cawthorne *et al.*, in preparation). Recent reviews of some of the results of very long baseline polarimetry have been provided by Wardle and Roberts (1988) and Roberts *et al.* (1990).

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FUNDAMENTAL RELATIONSHIPS

The measurement of linear polarization by an interferometer is most conveniently done by detection of both senses of circular polarization. In an obvious notation where the brackets denote a time average and the tilde denotes the visibility (as opposed to the sky) plane, the visibilities that correspond to the four Stokes parameters are sums and differences of the four complex cross-correlations that can be formed from the right- and left- circularly polarized electric fields detected at a pair of antennas labeled 1 and 2:

$$\tilde{I}_{12} = \frac{1}{2}\left(\langle E_{1R}E^*_{2R}\rangle + \langle E_{1L}E^*_{2L}\rangle\right) \ ,$$

$$\tilde{Q}_{12} = \frac{1}{2}\left(\langle E_{1L}E^*_{2R}\rangle + \langle E_{1R}E^*_{2L}\rangle\right) \ ,$$

$$\tilde{U}_{12} = \frac{i}{2}\left(\langle E_{1L}E^*_{2R}\rangle - \langle E_{1R}E^*_{2L}\rangle\right) \ ,$$

$$\tilde{V}_{12} = \frac{1}{2}\left(\langle E_{1R}E^*_{2R}\rangle - \langle E_{1L}E^*_{2L}\rangle\right) \ ,$$

The two components of the linear polarization (in either the visibility or the sky plane) are conveniently combined into a complex pseudo-vector P,

$$P = Q + iU = p\,e^{+2i\chi} = mI\,e^{+2i\chi} = M\,I \ .$$

Here the linearly polarized flux is $p = |P| = (Q^2 + U^2)^{1/2}$, the complex fractional linear polarization is $M = P/I = m\,e^{+2i\chi}$, the fractional linear polarization is $m = p/I$, and the position angle of the electric vector on the sky is $\chi = (1/2)\arctan(U/Q)$, measured east from north.

Total Intensity Distribution

The distribution of total intensity on the sky, $I(x,y)$, is related to the visibility $\tilde{I}_{12} = \tilde{I}(u,v)$ by the usual Fourier relationship for small fields of view,

$$\tilde{I}(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dx\,dy\,I(x,y)\,e^{+2\pi i(ux+vy)} \ ,$$

where u and v are the east and north components of the projection on the sky of the baseline connecting antennas 1 and 2. Thus *the total intensity distribution is the Fourier transform of the average of the RR and LL correlations.* In VLBI the circular polarization is often ignored because it is observed to be small in synchrotron sources, and $I(x,y)$ taken to be the transform of LL or RR alone, permitting observations to be made in a single polarization. Since $I(x,y)$ is a real quantity, the (u,v) point conjugate to $\tilde{I}(u,v)$ can be obtained from $\tilde{I}(u,v)$ simply by complex conjugation,

$$\tilde{I}(-u,-v) = \tilde{I}(u,v)^* \ ,$$

and "both sides" of the (u,v) plane filled in this way.

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Linear Polarization

The distribution of the (complex) linear polarization on the sky, $P(x,y)$, is related to its visibility $\tilde{P}(u,v)$ by

$$\tilde{P}(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dx\, dy\, P(x,y)\, e^{+2\pi i(ux+vy)} \ .$$

Combining the definitions of $\tilde{Q}$ and $\tilde{U}$ we see that the visibility for linear polarization is just

$$\tilde{P}(u,v) = \langle E_{1R}E^{*}_{2L}\rangle \ ,$$

so that *the polarization distribution is the Fourier transform of the RL correlation* (Conway and Kronberg 1969). However, the (u,v) point conjugate to $\tilde{P}(u,v)$ cannot be obtained from $\tilde{P}(u,v)$ because $P(x,y)$ is complex. Rather, it is given by the complex conjugate of the LR correlation,

$$\tilde{P}(-u,-v) = \langle E_{1L}E^{*}_{2R}\rangle^{*} \ .$$

Thus dual-polarization receivers are required at each station in order to "cover" the (u,v) plane.

SOME COMPLICATIONS

Real Feeds

No real antenna, feed, and receiver chain responds to only one sense of polarization, so we model the inevitable polarization contamination by introducing cross terms D that represent the fractional response to the orthogonal sense of polarization. In addition, we introduce complex antenna gains G that describe the amplitude and phase gains of a given chain. Thus we write the voltage detected in the right- and left-circularly polarized chains as

$$V_R = G_R\left(E_R e^{-i\phi} + D_R E_L e^{+i\phi}\right)$$

and

$$V_L = G_L\left(E_L e^{+i\phi} + D_L E_R e^{-i\phi}\right) \ .$$

Here the antenna gains for the right-circularly polarized antenna are

$$G_R = g_R\, e^{+i\psi_R} \ ,$$

where g_R and ψ_R are the amplitude and phase gains, and the Ds are the polarization contamination terms. The parallactic angle ϕ is the angle between the local vertical on the feed and the direction to the north at the position of the source on the sky. It is a constant for equatorially-mounted antennas, and varies with time for alt-azimuth mounts according to

$$\tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{\cos b \sin H}{\sin b\cos\delta - \cos b\sin\delta\cos H}$$

for a source at declination δ, observed at hour angle H by an antenna at latitude b.

Cross-Correlations

If we form the four cross-correlations of the voltages detected at the two antennas, the time averages may be expressed as:

$$R_1R_2^* \equiv \langle V_{R1}V_{R2}^*\rangle = G_{1R}G_{2R}^*(\tilde{I}_{12}+\tilde{V}_{12})e^{i(-\phi_1+\phi_2)} ,$$

$$L_1L_2^* \equiv \langle V_{L1}V_{L2}^*\rangle = G_{1L}G_{2L}^*(\tilde{I}_{12}-\tilde{V}_{12})e^{i(+\phi_1-\phi_2)} ,$$

$$R_1L_2^* \equiv \langle V_{R1}V_{L2}^*\rangle = G_{1R}G_{2L}^*\left[\tilde{P}_{12}e^{i(-\phi_1-\phi_2)}\right.$$

$$\left.+D_{1R}(\tilde{I}_{12}-\tilde{V}_{12})e^{i(+\phi_1-\phi_2)}+D_{2L}^*(\tilde{I}_{12}+\tilde{V}_{12})e^{i(-\phi_1+\phi_2)}\right] ,$$

$$L_1R_2^* \equiv \langle V_{L1}V_{R2}^*\rangle = G_{1L}G_{2R}^*\left[\tilde{P}_{12}^*e^{i(+\phi_1+\phi_2)}\right.$$

$$\left.+D_{1L}(\tilde{I}_{12}+\tilde{V}_{12})e^{i(-\phi_1+\phi_2)}+D_{2R}^*(\tilde{I}_{12}-\tilde{V}_{12})e^{i(+\phi_1-\phi_2)}\right] .$$

We have dropped terms second order in D or m since each is typically a few percent. The important result is that *the D terms enter the parallel-hands only as D^2 or Dm (neglected here), but contaminate the cross-hands as D.* For example, a feed with a 20 dB isolation in total power will lead to a 10 dB = 10% contamination of the cross-polarized output. Since $|P|$ is typically a few percent of $|I|$, this instrumental contamination is of the order of the expected polarized signal from the source, and it must be calibrated and removed.

Solution for Instrumental Parameters

The fringes are affected by time-varying antenna-based phases (the $\psi_i[t]$) from each end of the baseline. This is overcome in phase-connected interferometry by using observations of point calibrators. Unfortunately, this is not currently practical for VLBI because of the lack of suitable sources, and because of the rapid variability of atmospheric and clock contributions to the antenna phases. However, since the same phases affect both the parallel- and cross-hand correlations, they may be removed by dividing the cross-hand fringes by the parallel-hand fringes:

$$\frac{R_1L_2^*}{R_1R_2^*} = \frac{1}{r_2}e^{+i\Psi_2}\left[\tilde{M}_{12}e^{-2i\phi_2}+D_{1R}e^{+2i\delta_{12}}+D_{2L}^*\right] ,$$

$$\frac{R_1L_2^*}{L_1L_2^*} = r_1e^{+i\Psi_1}\left[\tilde{M}_{12}e^{-2i\phi_1}+D_{1R}+D_{2L}^*e^{-2i\delta_{12}}\right] ,$$

$$\frac{L_1R_2^*}{R_1R_2^*} = \frac{1}{r_1}e^{-i\Psi_1}\left[\tilde{M}_{12}^*e^{+2i\phi_1}+D_{1L}+D_{2R}^*e^{+2i\delta_{12}}\right] ,$$

$$\frac{L_1R_2^*}{L_1L_2^*} = r_2e^{-i\Psi_2}\left[\tilde{M}_{12}^*e^{+2i\phi_2}+D_{1L}e^{-2i\delta_{12}}+D_{2R}^*\right] .$$

We have neglected any circular polarization, and defined ratio of amplitude gains at one station as

$$r_1 = \frac{g_{1R}}{g_{1L}} ,$$

the difference of the right and left phase gains at one station (known as the "AC phase difference" at the VLA) as

$$\Psi_1 = \psi_{1R} - \psi_{1L} ,$$

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and the difference in the parallactic angles at the two stations as

$$\delta_{12} J = J\phi_1 - \phi_2 \ .$$

Here r, Ψ, and δ are all functions of time, with only $\delta(t)$ known *a priori*.

To find the Ds from these equations we must know the $r_i(t)$ and $\Psi_i(t)$. Fortunately, the ratio of parallel-hand fringes contains this information:

$$\frac{R_1 R_2^*}{L_1 L_2^*} = r_1 r_2 \, e^{+i(\Psi_1 - \Psi_2 - 2\delta_{12})} \ .$$

Using this relationship, the difference phases Ψ and the amplitude gain ratios r can be monitored as a function of time. As long as any circular polarization can be neglected, this relation may be applied to a resolved source. Using this information we are able to "line up" all the fringes, and solve the cross-to-parallel-hand ratios for the Ds; a reference antenna is required for R-L phases, but not for the R/L amplitudes. Calibration sources must have polarization structure that is unresolved ($\tilde{M}_{12} = \tilde{M}$), or better yet, be unpolarized ($\tilde{M}_{12} = 0$). We have found OQ 208 and 3C 84 to be suitable at 5 GHz. Values for D range from 0.5% to 15% (!), with 1–2% being typical. The effects of the Ds are then removed from the cross-fringes themselves. The final step in polarization calibration is to set the absolute position angle of the electric vectors on the sky, which requires observations of a source of known linear polarization. We typically use BL Lacertae objects such as 0300+470 or OJ 287 for this purpose.

Unlike the case for small arrays such as the VLA, where one D must be arbitrarily set to zero, in VLBI (using alt-azimuth mounted antennas) each of the Ds can be determined separately. This is because the two terms in D in each cross-to-parallel ratio are uncoupled by the difference in the parallactic angles of the feeds if the two antennas are located sufficiently far apart.

In these experiments, because of the small amplitudes of the cross-hand fringes, large antennas and wide recording bandwidths are required. Thus we have employed the Mark III data system, where the multiple tracks also provide the capability for simultaneous recording of two senses of circular polarization. Typically, we record seven 2 MHz channels each of right and left circular polarization in "mode C". This permits all of the parallel and cross-correlations to be formed by the Haystack Mark IIIA correlator. Parallel- and cross-hand correlations are done in separate passes because of the limited number of correlator modules that are available.

GLOBAL FRINGE FITTING FOR VLBP

The cross-polarized fringes are weak; typically,

$$|\mathrm{RL}| \sim (\text{few percent}) \times |\mathrm{RR}| \ .$$

VLBI fringes must be found in a search in delay–delay-rate space. The result is that, for a typical Mark IIIA search, fringes with signal-to-noise ratios less than 7 are likely to be noise peaks, and many weak cross-polarized fringes can be lost. *A priori* knowledge of the "location" of the cross-fringes would eliminate the requirement of a fringe search, and permit the recovery of cross-fringes of all signal-to-noise ratios. To overcome this limitation we have developed the following simple technique to predict the delay–delay-rate location of the cross-hand fringes from that of the parallel-hand fringes (Brown, Roberts, and Wardle 1989).

The delays in an interferometer are polarization dependent. The delay for correlator pq on baseline 12 may be written

$$\tau_{12}^{pq} = \tau_1^p - \tau_2^q + \epsilon_{12}^{pq} \ ,$$

where τ_1^p and τ_2^q are antenna-based terms, and ϵ_{12}^{pq} is a baseline-dependent term. We define the *delay offset* $\delta\tau$ for a given station to be the difference between the delays measured in right- and left-circular polarization,

$$\delta\tau_1 = \tau_1^R - \tau_1^L \ .$$

Neglecting the baseline terms, we can find the delay offsets for each antenna on a given baseline from combinations of parallel-hand and strong cross-hand fringes on that one baseline,

$$\delta\tau_1 = \tau_{12}^{RL} - \tau_{12}^{LL} = \tau_{12}^{RR} - \tau_{12}^{LR} \ ,$$

$$\delta\tau_2 = \tau_{12}^{RL} - \tau_{12}^{RR} = \tau_{12}^{LL} - \tau_{12}^{LR} \ .$$

The equations for the delay-rate offsets $\delta\dot{\tau}$ are analogous.

An offset for each antenna can be determined from baselines connecting it to every other antenna. These are found to be consistent, with typical values of 10Us of nanoseconds. The offsets are stable over the duration of an experiment (roughly a day), and any baseline-dependent terms are small. There are no significant polarization-dependent delay-rate offsets at 5 GHz.

Given the average delay offset $\langle\delta\tau\rangle$ for each station, the delay–delay-rate location of any cross fringe can be predicted from that of the parallel hand fringe by

$$\tau_{12}^{RL} = \tau_{12}^{LL} + \langle\delta\tau_1\rangle = \tau_{12}^{RR} + \langle\delta\tau_2\rangle \ ,$$

$$\tau_{12}^{LR} = \tau_{12}^{LL} - \langle\delta\tau_2\rangle = \tau_{12}^{RR} - \langle\delta\tau_1\rangle \ ,$$

$$\dot{\tau}_{12}^{RL} = \dot{\tau}_{12}^{LR} = \dot{\tau}_{12}^{RR} = \dot{\tau}_{12}^{LL} \ .$$

This technique can be tested using cross fringes from an unpolarized source. Since such fringes are determined entirely by the instrumental parameters, if the experiment can be calibrated using one source (3C 84), then the fringes on another (OQ 208) may be predicted independently. Such tests show that the recovery of cross-fringes of any signal-to-noise ratio can be accomplished in this simple manner.

IMAGING

I images are made by hybrid (self-calibration) techniques in the usual manner from the RR and LL fringes. An advantage of dual-polarization observations is that additional diagnostics, such as comparison of the RR and LL closure phases, are available. The corresponding *P* images are made using the antenna gains

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$G_{ip}(t)$ that are automatically found along with the I images. Alignment of the I and P images to within a small fraction of a synthesized beamwidth is assured by this use of common antenna gains. Making a complex P image (rather than separate real Q and U images) permits using "unmatched" cross-correlations (RL in the absence of LR, and *vice versa*) that may occur due to equipment limitations or malfunctions, or due to data processing difficulties. If, as a result, the (u, v) coverage is not symmetric, the dirty P beam will be complex, and a complex CLEAN is required. Because the total intensity distributions of most sources are dominated by a small number of compact regions that are easily imaged, the antenna phases will be well-determined, and the quality of the P image is rather insensitive to the details of the corresponding I image.

THE FUTURE OF VERY LONG BASELINE POLARIMETRY

The completion of the VLBA will usher in a new era in VLBI science. Improved (u, v) coverage and uniform calibration procedures will produce higher dynamic range polarization images. With the ability to observe more frequently, "magnetic movies" showing the evolution of magnetic fields and plasma in parsec-scale jets will become a reality within a few years. It will also be possible to observe the formation and evolution of shock-like structures in considerable detail, and to study the rapid (intra-day) polarization changes that have already been observed in at least one BL Lacertae object (Gabuzda, Wardle, and Roberts 1989b).

The multi-frequency capability of the VLBA will be crucial to polarization-sensitive observations, enabling one to explore through the Faraday effect the thermal plasma in and surrounding the radio emitting regions. Significant new information about the optical narrow line region and the relativistic electron spectrum should result. However, the use of non-VLBA antennas will be very important in the creation of the "scaled arrays" so necessary to determine the multi-frequency structure and polarization of radio sources. The use of large aperture non-VLBA antennas, such as the new Green Bank Telescope, will also be important for polarization observations of weaker sources. Finally, orbiting VLBI (even with a single hand of polarization on the spacecraft) will provide important information about the "cores" of the brightest objects, provided that ground stations observe with both hands of circular polarization.

New data processing techniques will also be important. In particular, simultaneous global fringe fitting of all four correlations (in AIPS) will reduce systematic errors present in the current data sets.

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REFERENCES

Brown, L. F., Roberts, D. H., and Wardle, J. F. C. 1989, *Ap. J.*, , **97**, 1522.
Conway, R. G., and Kronberg, P. P. 1969, *M.N.R.A.S.*, **142**, 11.
Cotton, W. D., *et al.* 1984, *Ap. J.*, **286**, 503.
Gabuzda, D. C. , Cawthorne, T. V., Roberts, D. H., and Wardle, J. F. C. 1989, in *BL Lac Objects, Lecture Notes in Physics*, **334**, ed. L. Maraschi, T. Maccacaro, and M.-H. Ulrich (Berlin: Springer-Verlag), p. 22.
Gabuzda, D. C., Wardle, J. F. C., and Roberts, D. H. 1989a, *Ap. J. (Letters)*, **336**, L59.
Gabuzda, D. C., Wardle, J. F. C., and Roberts, D. H. 1989b, *Ap. J.*, **338**, 743.
Roberts, D. H., Gabuzda, D. C., and Wardle, J. F. C. 1987, *Ap. J.*, **323**, 536.
Roberts, D. H., Kollgaard, R. I., Brown, L. F., Gabuzda, D. C., and Wardle, J. F. C. 1990, *Ap. J.*, **360**, 408.
Roberts, D. H., Potash, R. I., Wardle, J. F. C., Rogers, A. E. E., and Burke, B. F. 1984, in *IAU Symposium No. 110, VLBI and Compact Radio Sources*, ed. R. Fanti, K. Kellermann, and G.Setti, (Dordrecht: Reidel), p. 35.
Roberts, D. H., Wardle, J. F. C., Gabuzda, D. C., and Cawthorne, T. V. 1990, in *Parsec-Scale Radio Jets*, ed. J. A. Zensus and T. J. Pearson (Cambridge: Cambridge University Press), p. 110.
Wardle, J. F. C., and Roberts, D. H. 1988, in *IAU Symposium No. 129, The Impact of VLBI on Astrophysics and Geophysics*, ed. M. J. Reid and J. M. Moran, (Dordrecht: Reidel), p. 143.
Wardle, J. F. C., Roberts, D. H., Potash, R. I., and Rogers, A. E. E. 1986, *Ap. J. (Letters)*, **304**, L1.

Mel Wright: Is the relative positions of polarized and unpolarized flux determined by polarization VLBI observations?
D. H. Roberts: Since the same antenna phases are applied to both the parallel-hand and cross-hand fringes, any offset between the I and P images is a small fraction of a beamwidth. The absolute position angle of the electric vectors is set by observation of a compact highly-polarized source, typically a BL lac object.

Jerzy Usowicz: Is the point-spread function used in complex CLEAN positive definite? CLEAN converges only for positive definite psf's.
Dave Roberts: (Response provided by Ulrich Schwarz) Q and U are separately positive semi-definite because they are derived from the F.T. of V_Q and V_U (which are complex visibilities) with weights $\geq$ 0. Q and U may be negative, but this is no objection for CLEAN.

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