

THE MEASUREMENT OF DOMESTIC VENTILATION:

AN EXPERIMENTAL AND THEORETICAL INVESTIGATION WITH PARTICULAR REFERENCE TO THE USE OF CARBON DIOXIDE AS A TRACER SUBSTANCE*†

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(With Plate 1 and 4 Figures in the Text)

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1. INTRODUCTION

The post-war need for large numbers of dwellings has promoted renewed inquiry into the basic criteria by which good practice in the design and construction of buildings may be judged. More emphasis than ever before is now being put upon the functional characteristics of buildings regarded as living-spaces rather than as structures.

Among such functional characteristics few are more important than those physical attributes of rooms which may be collectively studied under the heading of room climatology, and which determine not only the thermal comfort of the occupants but also the state of the atmosphere investing them. Although the subject of ventilation, or the provision of a comfortable and healthful atmosphere, is inseparable from that of heating and their joint study has a long history, there are still many unsettled questions concerning the optimum amount (and especially the minimum amount) of air that it is desirable to supply. It is obvious that reliable methods of measuring ventilation are a prerequisite for the specification of optimum degrees of ventilation.

When mechanical or forced ventilation is employed it is often considered sufficient to state the total rate at which air is supplied to and extracted from the room, and this quantity is relatively easy to estimate without reference to its actual distribution within the room. With the natural ventilation that prevails in most private dwellings, such a simple estimation is neither possible nor desirable, for emphasis needs to be placed upon the ability of the ventilation to refresh the whole room and to prevent the accumulation of impurities released within it. Methods of measuring directly the rate at which impurities are swept away are therefore of great importance. This leads to a consideration of methods in which the rate of fall in concentration of an artificially introduced tracer substance is followed by continuous or repeated sampling, and which (subject to the maintenance of adequate mixing) ultimately yield a specification of the overall rate of air change in the room. The principal questions now arising concern, firstly, the choice of tracer substance and, secondly, the method to be employed in assaying that substance.

The whole technique should be one that can be used in ordinary living rooms without interfering with the normal conduct of routine household tasks by the occupants, and preferably also without detriment to any collateral investigations into their physiological reactions. It is clear that to satisfy these requirements the tracer substance must be hygienically safe, technically hazardless, and measurable to a sufficient accuracy by means of apparatus that is simple, standardized and readily procurable.

The method in which carbon dioxide is estimated

by means of the standard Haldane gas analysis apparatus meets these conditions to a large extent and has the added advantage that the apparatus may also be used to study the respiratory exchange of the room occupants during determinations of the energy expended in domestic work. Although the method cannot be described as new in conception, since many previous workers have used carbon dioxide as a tracer and have estimated its concentration by volumetric analysis, there still appears to be considerable scope for improvement in the regularization of the procedure in its application to ventilation measurement.

Accordingly, the present paper records an attempt to define a standard technique for the estimation of rates of air change in occupied and unoccupied rooms using carbon dioxide as a tracer gas, to analyse theoretically and experimentally the various factors concerned, and to determine the more important sources of error inherent in the method.

2. HISTORICAL REVIEW

More than a century ago Tredgold (1824) showed that the relationship between the concentration of CO₂ in the air of a room and the rate of ventilation could be defined if the amount of CO₂ expired by the occupants was taken into account. Max von Pettenkofer (1858) appears to be the first worker to introduce a suitable experimental technique, consisting in the main of two methods. In the first CO₂ was produced from a more or less steady source—human respiration—and the air change was estimated from the final CO₂ concentration when equilibrium had been reached. The second method consisted in the liberation of a quantity of CO₂ from the burning of standard candles or by a charcoal fire, and estimating the rate of decay of the gas.

Petri (1889) suggested that a higher CO₂ concentration than that used by Pettenkofer would increase the accuracy of the method, and employed a cylinder of the compressed gas. This method was used by early German workers, e.g. Wolpert (1905) and Kisskalt (1913). Haldane & Osborn (1902) generally used standard candles and this source of CO₂ was used recently by the Building Research Station (1927). Crowder (1911), working on the ventilation of Pullman carriages, estimated the ventilation rate from the decline of CO₂ which had accumulated during the passage of a train through a tunnel. Masterman, Dunning & Densham (1935), in a study of flueless gas heaters, used the CO₂ generated in the room by the appliance. Recent workers using CO₂ from cylinders are Ilzhöfer (1930), Masterman *et al.* (1935), Süpfle (1936), Warner (1940) and Carne (1946).

The use of water vapour as a tracer gas was suggested by Houghton & Blackshaw (1933), who

found it easier to use and more accurate than the Pettenkofer CO₂ method. However, Dufton & Marley (1934) found the method unsuitable owing to absorption of water vapour by walls and furnishings, and a preliminary investigation by ourselves led to similar conclusions.

Marley (1935) used hydrogen and estimated it by means of a katharometer. Warner (1940) and Bedford, Warner & Chrenko (1943) used the same method with coal gas as the source of hydrogen. Recently, Lidwell & Lovelock (1946) described a number of organic substances suitable as tracers and stressed the value of tertiary butyl hypochlorite and a simple estimation by *o*-tolidine and benzoic acid. In our hands, the method did not give consistent results and was eventually discarded.

3. THEORETICAL ANALYSIS

3.1. *Infiltrative ventilation contrasted with perfative ventilation*

Ventilation is, in essence, a process by which room air, contaminated by the gaseous (and possibly also particulate) products of operations conducted in the room, is replaced by fresh air from outside. From this standpoint, the efficacy of ventilation clearly depends not so much on the detailed mechanism by which the fresh air is introduced and the vitiated air extracted, as upon the thoroughness with which the incoming air permeates all parts of the room to sweep away lingering impurity.

When air is admitted and abstracted through large un baffled vents, when windows or doors are wide open, or when strong draughts are induced by heated flues, a room is likely to be traversed by currents of air that largely preserve their identity between inlet and outlet, and in these circumstances the velocity of the currents may in itself form an appropriate measure of the degree of ventilation.

In contradistinction to such a mode of perfation or through-ventilation, there are many occasions in the realms both of 'natural' ventilation and of judiciously applied artificial ventilation, when the air currents do not preserve their identity during their traverse, but mingle, by a process of eddy diffusion, more or less intimately with the existing air in the room. A quantity of room air, equivalent to that which has entered, is meanwhile expelled to the external atmosphere through whatever apertures may be available. This process is typically exemplified by the 'natural' ventilation of an almost 'closed' room, which takes place by the infiltration and exfiltration of air through cracks, chinks and even pores in the otherwise solid material of the walls. Unlike the more drastic through-ventilation, the process now considered may be divided into three phases: first, the infiltration of external air into the room; secondly, the mixing of this infiltrated air with

the prevailing air content of the room; and thirdly, the exfiltration of an equivalent volume of the room mixture. These three phases, however, occur simultaneously rather than successively in time; and the significant characteristic of the process is complete mixing of the air content of the room which ensures that the outgoing air shall possess the same composition as the air remaining in the room at that instant. Although this type of exchange will usually be associated with low rates of ventilation such as occur in 'closed' rooms, this is by no means necessarily so, for sufficiently vigorous stirring by fans can produce and maintain complete mixing of the room air even when the inlet and outlet apertures are wide open and therefore conducive to direct draughts. It is evident, especially when mechanical stirring is employed, that, although they may be influential in determining an occupant's sensation of freshness, the velocities of the air currents in the room bear no necessary relation to the rate at which fresh air is entering and vitiated air is leaving, and do not suffice to specify the degree of ventilation. Under such conditions, however, the conventional 'rate of air change' purports to give a more or less direct measure of the rapidity of evacuation of vitiating products. The postulate of perfect mixing will be accepted as a basis for the ensuing theory of this measure, discussion of the practical difficulties of realization being postponed until § 3.8 and brought out more fully in the experimental work (§ 5D).

A simple theory of ventilation based (perhaps somewhat implicitly) on the concept of perfect mixing has long been current and forms the basis of most statements about the rates of air change experienced in rooms. Ventilation rates are often thus calculated in circumstances which would not be expected to afford formal justification for the hypothesis of perfect mixing, but experience nevertheless shows that the approximation is reasonably satisfactory over a remarkably wide range of actual conditions.

The origin of the theory of complete mixing applied to infiltrative ventilation can be traced back at least as far as Thomas Tredgold (1824), who, in the second edition of his text-book, gave an addendum to Chapter IV entitled 'On the alteration of the proportion of the components of atmospheric air by want of ventilation', which contains what is tantamount to a finite-difference treatment of an exchange problem that nowadays would be handled by means of the differential calculus.

Without attempting any further survey of the historical development of the subject, it is now proposed to discuss this classical ventilation process with somewhat more generality than has been customary, and to examine the situation in which any given constituent (including impurities under the heading of constituents) of the air of a room is

supposed to be present already in arbitrary concentration in the infiltrating air and to be produced continuously within the room at any specified rate. Account will also be taken of the possibility of absorptive and diffusive loss, of possible differences between the temperature and pressure inside and outside the room, and of time variations occurring in any of these quantities. The object of the analysis is to establish a mathematical model to explain the time variation in the concentration of constituents such as CO₂, water vapour, hydrogen, helium, etc., which may be produced, for example, by the respiration of human occupants and the combustion of fuels, or which may have been introduced deliberately to act as 'indicators' or tracer substances for the purpose of measuring ventilation.

3.2. The differential equations of transfer

Consider a room of volume V in which a specified constituent of the air, say the s th, is being generated by some source at a mass rate K_s ; and suppose that this constituent, like all the others, is being continuously and completely mixed not only with the air already in the room but also with the incoming air. Let the temperature and pressure within the room be T, P , while in the atmosphere outside they are T', P' . Suppose that the external air is entering the room at a total volume rate Q' (measured at T', P'), while the internal air is leaving the room at a corresponding total volume rate Q (measured at T, P). Let the absolute concentration or partial density (i.e. the mass per unit volume of space occupied) of the constituent s be ρ_s within the room and ρ'_s in the air infiltrating from outside.

Then considering the constituent s within the room, it is seen that the rate of mass influx is $Q'\rho'_s$ and the rate of mass efflux is $Q\rho_s$, while the rate of mass emission within the room is K_s . The mass of the constituent actually in the room at any instant is

$V\rho_s$ and its rate of increase is $V\frac{d\rho_s}{dt}$, where t denotes time. Thus by the conservation of mass,

$$V\frac{d\rho_s}{dt} = K_s + Q'\rho'_s - Q\rho_s, \tag{1}$$

with precisely similar continuity equations for all other constituents of the air.

If all constituents are assumed to behave like perfect gases, the laws

$$\rho_s = \frac{M_s P_s}{RT} = \frac{M_s P}{RT} p_s; \quad \rho'_s = \frac{M_s P'_s}{RT'} = \frac{M_s P'}{RT'} p'_s \tag{2}$$

will be obeyed, where P_s and P'_s are respectively the partial pressures, in the room air and incoming air, of the component s of molecular weight M_s , R is the absolute gas constant (equal to 1545 ft.lb./lb.mol./°F.) and

$$p_s \equiv P_s/P; \quad p'_s \equiv P'_s/P' \tag{3}$$

are the corresponding proportional partial pressures or relative concentrations.

Substitution of (2) in (1) gives

$$V\frac{d}{dt}\left(\frac{P}{T}p_s\right) = R\frac{K_s}{M_s} + Q'\left(\frac{P'}{T'}p'_s\right) - Q\left(\frac{P}{T}p_s\right),$$

from which

$$\frac{dp_s}{dt} + \left(\frac{Q}{V} + \frac{d}{dt}\log_e\frac{P}{T}\right)p_s = \frac{T'}{VP}\left(R\frac{K_s}{M_s} + Q'\frac{P'}{T'}p'_s\right). \tag{4}$$

For practical application this equation is perhaps more convenient than (1), since the proportional partial pressures p_s, p'_s are identical with the volumetric proportions directly disclosed by ordinary gas analysis, whereas the partial densities ρ_s, ρ'_s are rarely measured. Before attempting to integrate the equations typified by (4), it is necessary to examine the terms on the right-hand side. For any constituent of low concentration p'_s in the external air, the rate of change of room concentration will be determined more by Q than by Q' , and it is therefore preferable to express the latter in terms of the former.

Now by virtue of Dalton's law of partial pressures, it is obvious from (3) that

$$\Sigma p_s = 1 = \Sigma p'_s,$$

so that summation of equations (4) for all constituents s gives

$$\frac{Q}{V} + \frac{d}{dt}\log_e\frac{P}{T} = \frac{T'}{VP}\left(R\Sigma\frac{K_s}{M_s} + Q'\frac{P'}{T'}\right),$$

whence the relation between Q' and Q is found to be

$$Q' = \frac{PT'}{P'T}\left(Q - \frac{RT}{P}\Sigma\frac{K_s}{M_s} + V\frac{d}{dt}\log_e\frac{P}{T}\right). \tag{5}$$

It is implicit in this relation that the entering air Q' assumes the room temperature T when it mixes with the room air. Insertion of (5) in (4) yields

$$\frac{dp_s}{dt} + \left(\frac{Q}{V} + \frac{d}{dt}\log_e\frac{P}{T}\right)(p_s - p'_s) = \frac{RT}{VP}\left(\frac{K_s}{M_s} - p'_s\Sigma\frac{K_s}{M_s}\right) \tag{6}$$

as the differential equation determining the variation of the proportion p_s of the constituent s in the room air.

Since the present treatment, unlike that usually encountered in the literature, has distinguished between the volume rate of inflow Q' and the volume rate of outflow Q , it is legitimate to identify not the ratio Q'/V but the ratio

$$q \equiv Q/V \tag{7}$$

with what is customarily called the rate of air change expressed as the number of room volumes of room air (measured at room temperature and pressure) expelled in unit time. It is convenient also to express

the rate of production K_s of the constituent s in terms of the equivalent number

$$v_s \equiv \frac{RT}{VP} \frac{K_s}{M_s}, \tag{8}$$

of room volumes (measured at room temperature and pressure) emitted in unit time, so that (6) becomes

$$\frac{dp_s}{dt} + \left(q + \frac{d}{dt} \log_e \frac{P}{T} \right) (p_s - p'_s) = v_s - p'_s \Sigma v_s. \tag{9}$$

3.3. Variation of emission with concentration

It is desirable to be able to treat the net rate of emission K_s , and hence also v_s , as potentially dependent upon the concentration p_r of any arbitrary constituent r of the air. There appear to be three distinct sources for such dependency. First, the rate of generation of certain constituents may be directly affected by the partial pressure not only of themselves but also of other constituents present in the air. Thus, for example, as discussed further in §3.7, the oxygen consumption and output of CO_2 and water vapour by human beings is expected to be slightly dependent on the degree of respiratory embarrassment engendered by breathing air containing an abnormal proportion of CO_2 . Similarly, any flame or combustion appliance may remove a combustible constituent from the air, and emit its oxidation product, at a rate markedly dependent upon the ambient concentration of that constituent. Secondly, substances such as CO_2 and water vapour may be subject to absorption by walls, furnishings or liquids at a rate which is likely to be proportional to the difference $(P_s - P'_s)$ between their existing partial pressures and some saturation partial pressure P'_s associated with the absorbent.* Thirdly, true diffusion through permeable walls may be expected to remove any constituent from the room at a rate proportional to the difference $(P_s/\sqrt{T} - P'_s/\sqrt{T'})$ between its temperature-weighted partial pressures inside and outside the room. Since all three processes occur independently of the general air flow and would persist even if Q and Q' vanished, they may be termed non-ventilative and are appropriately treated as disturbances of the net rate of emission v_s .

The linear character of equation (9) can still be preserved if v_s is merely a linear function of p_s , and in order to retain a reasonable tractability in the solution, only this type of variation will be considered. This is tantamount to asserting that the first process above mentioned, like the second two, is linear in p_s , and so, by symmetry, linear in the

* A mechanism such as this appears capable of dealing with the condensation of water vapour on surfaces cooled below the dew-point, and also with the evaporation of water vapour from free water surfaces: In this case the coefficients of equation (10) would depend somewhat on the intensity of air movement in the room.

general $p_r (r \neq s)$ also. Without further loss of generality, it is now possible to write the rate of emission as

$$v_s = a_s + b'_s p'_s + \Sigma^r b_{sr} p_r, \tag{10}$$

where the coefficients a_s , b'_s and b_{sr} are independent of p_r and p'_s , but may vary with temperature and hence or otherwise with time. The set of coefficients b_{sr} describes the influence of the ambient concentration of any constituent r on the rate of emission of any constituent s , whether the same or different; the summation applies to the suffix r only.

By means of (10), equation (9) may be written in the form

$$\begin{aligned} \frac{dp_s}{dt} + p_s \left(q + \frac{d}{dt} \log_e \frac{P}{T} - b_{ss} + p'_s \Sigma^r b_{rs} \right) \\ = a_s + p'_s \left\{ q + \frac{d}{dt} \log_e \frac{P}{T} + b'_s - \Sigma^s (a_s + b'_s p'_s) \right\} \\ + \Sigma^{r \neq s} p_r (b_{sr} - p'_s \Sigma^a b_{ar}), \end{aligned} \tag{11}$$

which is a first-order differential equation, with (in general) variable coefficients, defining the time variation of the relative concentration p_s . Similar equations apply to all the constituents s . The symbol placed above each Sigma indicates the suffix with respect to which the summation is to be performed.

The outer summation in the last term of the right side of (11) extends over all constituents r except the s th, and indicates by the presence of p_r that the s equations like (11) are not independent but form a simultaneous set which strictly cannot be solved separately. The concentrations of different constituents are, in short, coupled by their effects on each other's rates of generation, and their time variations will be defined independently only by means of a differential equation of order equal to the number thus mutually coupled. This difficulty is, however, quite academic from the standpoint of practical application to ventilation, since the coefficients b_{sr} concern minor variations in a rate of emission v_s which is itself normally quite small, and a much more serious effect due to the ventilative coupling between adjacent rooms will presently be encountered in §3.10. Accordingly, the solution given will tacitly ignore the presence of p_r in the right side of (11) and the term $\Sigma^{r \neq s} p_r (b_{sr} - p'_s \Sigma^a b_{ar})$ will be neglected.

When applied to the calculation of rates of air change from experimental data, equation (11) does not require integration, as will be seen in the ensuing §3.5 (equation (24)); but the integrated version is obviously necessary for the inverse process of calculating the concentration at any instant from a knowledge of the rate of air change. Formal solution of (11) is relatively easy if the rate of air change q is independent of the concentration p_s of the indicator used, for then the equation is linear and

the solution is simply that given in §3.4 (equation (16)) below. The experimental success of q as a stable index of ventilation attests the fact that q is relatively insensitive to variations of p_s , but there are nevertheless physical reasons for suggesting that the invariance cannot be perfect, and hence that equation (11) is not absolutely linear. This question is again referred to in §3.9. In the meantime, §3.4 proceeds as though q were substantially independent of p_s .

3.4. Integration of the equations

If a mean modified rate of air change is defined as

$$q_s \equiv \frac{1}{t} \int_0^t \left(q - b_{ss} + p'_s \sum^r b_{rs} \right) dt, \tag{12}$$

while, for brevity,

$$A_s \equiv a_s + p'_s \left\{ q + \frac{d}{dt} \log_e \frac{P}{T} + b'_s - \sum^s (a_s + b'_s p'_s) \right\}, \tag{12}$$

and
$$B_s \equiv \log_e \frac{P}{T} + q_s t, \tag{13}$$

equation (11), denuded of its last term, contracts to

$$\frac{dp_s}{dt} + p_s \frac{dB_s}{dt} = A_s, \tag{14}$$

and, subject to B_s and A_s being independent of p_s , has the solution

$$p_s = e^{-B_s} \left(C + \int_0^t A_s e^{B_s} dt \right). \tag{15}$$

From the initial conditions

$p_s = p_{s0}$, $p'_s = p'_{s0}$, $T = T_0$, $P = P_0$, when $t = 0$, the constant C is found to equal $P_0 p_{s0} / T_0$, the solution becoming

$$p_s = \frac{T}{P} e^{-q_s t} \left\{ \frac{P_0}{T_0} p_{s0} + \int_0^t \frac{A_s P}{T} e^{q_s t} dt \right\}. \tag{16}$$

This expression enables formal account to be taken of arbitrary time variations in any of the basic quantities T , P , q , p'_s , a_s , b_{sr} and b'_s , and numerical evaluation of the integral can always be performed for practical data. The variations likely to be most important are those occurring in the ventilation rate q and the temperature T . Unfortunately, however, there is a severe limitation on the types of time variation for which explicit integration is possible in terms of known functions, and the solutions even then look cumbersome, so that it does not seem profitable to quote them in the present context.

If the parameters T , P , q , p'_s , a_s , b_{sr} and b'_s are all constant, equations (12) become

$$\left. \begin{aligned} q_s &\equiv q - b_{ss} + p'_s \sum^r b_{rs}, \\ A_s &\equiv a_s + p'_s \left\{ q + b'_s - \sum^s (a_s + b'_s p'_s) \right\} \\ &\equiv a_s + p'_s \left\{ q_s + b_{ss} + b'_s - \sum^r (a_r + b'_r p'_r + b_{rs} p'_s) \right\} \end{aligned} \right\} \tag{17}$$

while (16) reduces to

$$p_s = p_{s0} e^{-q_s t} + \frac{A_s}{q_s} (1 - e^{-q_s t}),$$

and can therefore be written exclusively in terms of the modified rate of air change q_s defined by (17) and a modified volume rate of emission

$$\begin{aligned} w_s &\equiv A_s - p_s - p'_s q_s \\ &\equiv a_s + p'_s \{ b_{ss} + b'_s - \sum^r (a_r + b'_r p'_r + b_{rs} p'_s) \}, \end{aligned} \tag{18}$$

in the form

$$p_s = p'_s + \frac{w_s}{q_s} + \left(p_{s0} - p'_s - \frac{w_s}{q_s} \right) e^{-q_s t}. \tag{19}$$

This type of equation is familiar in the literature of ventilation,* and in its logarithmic form (see the ensuing equation (23)) is identical with the formula attributed† to Hagenbach. The corresponding formula with w_s omitted is likewise attributed† to Seidel. References abound to the simple form with w_s and p'_s both omitted. The present somewhat lengthy derivation is justified by the detailed disclosure of the manner in which the various correction factors enter into the parameters w_s and q_s , and by the provision of means (represented by equation (16)) for dealing with non-steady conditions. A simple physical interpretation of the modified parameters w_s and q_s is somewhat difficult to state. They are essentially defined by (17) and (18) in terms of the coefficients a_s , b_{sr} and b'_s of expression (10), which describes how the net rate of emission within the room is influenced by the interior and exterior concentrations p_s and p'_s .

It is apparent from (19) that the concentration p_s of the constituent s subsides asymptotically from its initial value p_{s0} towards an equilibrium value

$$p_{s\infty} = p'_s + w_s / q_s, \tag{20}$$

at which the sum of the mass rates of efflux, absorption and diffusion counterbalances the sum of the mass rates of influx and generation. The rate of evanescence at any time t is

$$\frac{dp_s}{dt} = \{ w_s - q_s (p_{s0} - p'_s) \} e^{-q_s t} \tag{21}$$

and has an initial value

$$\left(\frac{dp_s}{dt} \right)_0 = w_s - q_s (p_{s0} - p'_s). \tag{22}$$

Initially, therefore, the concentration will increase or decrease according as $(p_{s0} - p'_s)$ is less or greater than w_s / q_s .

* See, for example, Masterman, Dunning & Densham (1935, p. 72), and various references cited therein. This publication also contains an interesting wealth of data on rates of air change in rooms of various sizes.

† C. Flügge (1881, pp. 487 et seq.). Flügge deduces Seidel's formula but merely quotes Hagenbach's formula without deduction.

3.5. Calculation of the rate of air change

Equation (19) shows that the quantity

$$(p_s - p'_s - w_s/q_s)$$

decays exponentially and implies the linear relation

$$y \equiv \log_e \left(p_s - p'_s - \frac{w_s}{q_s} \right) = \log_e \left(p_{s0} - p'_s - \frac{w_s}{q_s} \right) - q_s t \quad (23)$$

between y and the time t , the slope being equal to $(-q_s)$ and thus disclosing the rate of air change

$$q = q_s + b_{ss} - p'_s \sum b_{rs}$$

Unfortunately, it is obviously impossible to calculate y from the observed values p_s unless q_s is already known. Many experimenters, however, have ignored w_s/q_s or worked in circumstances in which it was negligible, and have found the graph of $\log_e (p_s - p'_s)$ versus time to be sufficiently straight for its slope to yield q_s unequivocally. More generally, if w_s is of appreciable but known magnitude, a plausible value of q_s may be guessed and used to derive an approximate value of y from p_s . From the slope of the resulting approximately straight graph an improved value of q_s may be obtained, which may be used iteratively to refine the values of y , to draw a straighter graph, and hence to derive ultimately the correct value of q_s . Unless the initial estimate of q_s is grossly in error, or the data p_s happen to be seriously erratic, the iterative process is usually rapidly convergent; but the procedure remains distinctly inconvenient and can be avoided.

In principle, q_s can also be found directly and simply from equations (20) or (22). The former suffers from the disadvantages that it is rarely convenient to wait long enough for a steady value of $p_{s\infty}$ to be attained and that w_s is usually too small to provide sufficient sensitivity; the latter equation involves the initial rate of decay $(dp_s/dt)_0$ which is difficult to determine accurately.

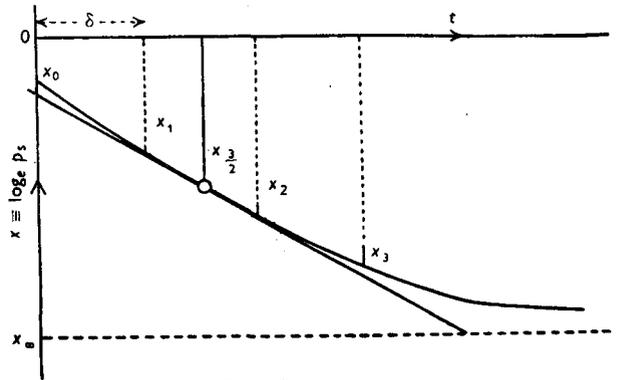
The alternative and much preferable procedure is to find the rate of air change q directly from the general unintegrated equation (9) which can be solved to give

$$q = - \left(1 + \frac{p'_s}{p_s - p'_s} \right) \frac{d}{dt} \log_e p_s + \frac{v_s - p'_s \sum v_s}{p_s - p'_s} + \frac{d}{dt} \log_e \frac{T}{P} \quad (24)$$

Unlike equations (17)–(23) inclusive, this result is neither contingent upon the constancy of any of its parameters nor even upon the independence of q and p_s . If, indeed, q is approximately constant and p'_s and v_s are small, as they usually are in ordinary ventilation experiments, the graph of $\log_e p_s$ against time will be almost linear, and its slope at once gives a good first approximation to the value of q . Without requiring any resort to initial approximate estimates of q , however, equation (24) enables a direct calculation to be made of the influence on q of the correction

terms due to p'_s , v_s and to changes in P and T . It is, in fact, clear that such corrections must necessarily be made before variations in q can justly be held responsible for departures of the graph from linearity. Any meticulous application of the corrections should take cognizance of the possible sources of variation of v_s indicated by equation (10).

As shown in an exaggerated form in the adjoining Text-fig. 1, when q is constant, a finite concentration p'_s in the incoming air and a positive net rate of generation v_s within the room both progressively diminish the slope of the $\log_e p_s$ graph and render it concave (since the experimental values of $\log_e p_s$ must be negative) to the time axis. Increase of the ratio T/P for the room also reduces the slope but (unless p'_s be large) not progressively.



Text-fig. 1.

The instantaneous rate of air change can be calculated readily from concentration-time data by means of equation (24), provided that the rate of emission v_s of the tracer substance is known. Occasions may arise, however, when the magnitude of v_s , although sensibly constant, is completely unknown, as, for example, in an auditorium or any room containing a concealed source. It then becomes necessary to adopt a procedure, such as the following, whereby v_s can be eliminated.

If, neglecting the minor variations indicated by (10), the parameters v_s , q , $\frac{d}{dt} \log_e \frac{P}{T}$ and p'_s are assumed constant, particularization of equation (9) for two distinct times denoted by the suffixes 1 and 2 gives

$$\begin{aligned} \left(\frac{dp_s}{dt} \right)_1 + \left(q + \frac{d}{dt} \log_e \frac{P}{T} \right) p_{s1} &= v_s + p'_s \left(q + \frac{d}{dt} \log_e \frac{P}{T} - \sum v_s \right) \\ &= \left(\frac{dp_s}{dt} \right)_2 + \left(q + \frac{d}{dt} \log_e \frac{P}{T} \right) p_{s2} \end{aligned}$$

Hence, by eliminating v_s , the rate of air change is found to be

$$q = \frac{1}{p_{s1} - p_{s2}} \left\{ \left(\frac{dp_s}{dt} \right)_2 - \left(\frac{dp_s}{dt} \right)_1 \right\} + \frac{d}{dt} \log_e \frac{T}{P} \quad (24a)$$

Also, by eliminating q ,

$$v_s - p'_s \Sigma v_s = \frac{1}{p_{s1} - p_{s2}} \left\{ (p_{s1} - p'_s) \left(\frac{dp_s}{dt} \right)_2 - (p_{s2} - p'_s) \left(\frac{dp_s}{dt} \right)_1 \right\} \quad (24b)$$

so that if p'_s is small, the rate of emission of v_s can also (at least in principle) be deduced from the measurements. Both these formulae depend on an accurate assessment of the difference between the slopes of the concentration-time curve at two points, an operation of some difficulty unless the data are very regular.

3.6. Graduation of experimental data

In order to determine q satisfactorily from (24), it is necessary to replace the raw data p_s , which inevitably exhibit some degree of experimental scatter, by a suitable graduation formula from which smoothed values of p_s and $\frac{d}{dt} \log_e p_s$ can be derived by interpolation and differentiation.

For most practical cases, the time graph of

$$x \equiv \log_e p_s \quad (25)$$

is only slightly curved and may be adequately represented by a quadratic function

$$x = \alpha + \beta t + \gamma t^2 \quad (26)$$

Now if there are $(N + 1)$ data $x_n (n = 0, 1, 2, \dots, N)$ corresponding to the times t_n , and provided that N is not less than 3, optimum values for the constants α, β, γ can be determined by minimizing the residual variance of x .* If

$$\bar{z} \equiv \frac{1}{N + 1} \sum_{n=0}^{n=N} z_n \quad (27)$$

* This criterion receives formal justification if the errors of x are independent of x . If, on the contrary, it is the errors in p_s that are independent of p_s , then in principle it would be preferable to minimize the residual variance of p_s rather than that of x . This could be achieved approximately by minimizing the mean square of the product of p_s and the residual of x , but the computational labour, and the difficulty of gauging the variance of the predicted value of x , would be much increased. For practical purposes, however, the distinction scarcely matters, and the chief requirement is a simple reproducible procedure. The method adopted has the advantage of yielding tractable equations enabling the constants of equation (26), and the variance of the resulting estimate of x , to be determined explicitly by algebraic methods.

denotes the mean value of any symbol z , the resulting equations for α, β, γ are

$$\left. \begin{aligned} \bar{x} &= \alpha + \beta \bar{t} + \gamma \bar{t}^2, \\ \bar{xt} &= \alpha \bar{t} + \beta \bar{t}^2 + \gamma \bar{t}^3, \\ \bar{xt}^2 &= \alpha \bar{t}^2 + \beta \bar{t}^3 + \gamma \bar{t}^4, \end{aligned} \right\}$$

on the assumption that all irregularities of the data are attributable solely to errors in x_n , the times t_n being measured without sensible error. The solutions of these equations, inserted in (26) and simplified by adoption of the abbreviations

$$\left. \begin{aligned} d_1 &\equiv \bar{t}^4 - \bar{t}^3, & d_2 &\equiv \bar{t}^3 - \bar{t}^2, & d_3 &\equiv \bar{t}^2 - \bar{t}, \\ d_4 &\equiv \bar{t}^4 - \bar{t}^2, & d_5 &\equiv \bar{t}^3 - \bar{t}, \\ d_6 &\equiv \bar{t}^2 - \bar{t}^2, & & & & \end{aligned} \right\} \quad (28)$$

and $D \equiv d_5 d_6 - d_6^2$,

give the graduation formula, for the value of x at any time t , in the form

$$x = \frac{1}{(N + 1) D} \sum_{n=0}^{n=N} x_n \{ (d_1 + d_2 t_n + d_3 t_n^2) + (d_2 + d_5 t_n + d_6 t_n^2) t + (d_3 + d_6 t_n + d_9 t_n^2) t^2 \}, \quad (29)$$

whence p_s is obtained by evaluating e^x .

Differentiation of (29) gives

$$\frac{dx}{dt} = \frac{1}{(N + 1) D} \sum_{n=0}^{n=N} x_n \{ (d_2 + d_5 t_n + d_6 t_n^2) + 2(d_3 + d_6 t_n + d_9 t_n^2) t \}, \quad (30)$$

which completes the material required for calculating q at any instant t from equation (24).

Both equations (29) and (30) are linear in the presumably uncorrelated data x_n . Thus by virtue of the well-known theorem that for any linear function

$$z = \Sigma E_n x_n,$$

where E_n is any coefficient independent of x_n and the x_n are uncorrelated, the variance is

$$\text{var } z = \Sigma E_n^2 \text{ var } x_n, \quad (31)$$

it will be seen that x and dx/dt have respectively the variances

$$\text{var } x = \frac{1}{(N + 1)^2 D^2} \sum_{n=0}^{n=N} \text{var } x_n \{ (d_1 + d_2 t_n + d_3 t_n^2) + (d_2 + d_5 t_n + d_6 t_n^2) t + (d_3 + d_6 t_n + d_9 t_n^2) t^2 \}^2 \quad (32)$$

and

$$\text{var } \frac{dx}{dt} = \frac{1}{(N + 1)^2 D^2} \sum_{n=0}^{n=N} \text{var } x_n \{ (d_2 + d_5 t_n + d_6 t_n^2) + 2(d_3 + d_6 t_n + d_9 t_n^2) t \}^2 \quad (33)$$

If, as seems very likely from the nature of the measurements, the errors in p_s have a Gaussian

distribution, with a variance that is small compared with, and independent of, the mean \bar{p}_s , then approximately from (25),

$$\text{var } x_n \doteq p_m^{-2} \text{var } p_{sn}. \tag{34}$$

By means of equation (24) it is thus possible to trace the effect on q of errors in measuring p_s . If the errors of p_s are regarded as the sole source of the variance of q , and p'_s is small compared with p_s , then for the conditions obtaining in all ordinary ventilation experiments the variance of q itself will be substantially given by $\text{var } (dx/dt)$.

If $\text{var } p_s$ is estimated solely from the residual variance $(\Delta p_{sn})^2$ between the data p_{sn} and the values e^{x_n} predicted by the graduation curve, allowance must be made for the loss of three degrees of freedom caused by fitting the constants α, β, γ , so that the 'internal variance', referred to in §5C, becomes

$$\text{var } p_s = \frac{N+1}{N-2} \overline{(\Delta p_{sn})^2},$$

where $\Delta p_{sn} \equiv p_{sn} - e^{x_n}$.

Should the experimental data indicate that the various corrections involved in (24) are negligible, a linear graduation function

$$x = \alpha + \beta t, \tag{26a}$$

implying truly exponential decay of p_s , may replace (26), when the results corresponding to (29) and (30) become explicitly

$$x = \frac{1}{(N+1) \overline{(t^2 - \bar{t}^2)}} \sum_{n=0}^{n=N} x_n \{(t^2 - \bar{t}^2) t_n + (t_n - \bar{t}) t\}, \tag{29a}$$

$$\frac{dx}{dt} = \frac{1}{(N+1) \overline{(t^2 - \bar{t}^2)}} \sum_{n=0}^{n=N} x_n (t_n - \bar{t}), \tag{30a}$$

the latter being independent of t . Since in this instance only two degrees of freedom have been lost by fitting the constants α, β ,

$$\text{var } p_s = \frac{N+1}{N-1} \overline{(\Delta p_{sn})^2},$$

and the variance of (30a) may be written

$$\text{var } \frac{dx}{dt} = \frac{\overline{(\Delta p_{sn})^2} \sum_{n=0}^{n=N} p_m^{-2} (t_n - \bar{t})^2}{(N^2 - 1) \overline{(t^2 - \bar{t}^2)^2}}. \tag{33a}$$

When the data x_n are obtained at regular time intervals δ , so that

$$t_n = n\delta, \tag{35}$$

the mean values can be expressed explicitly in terms of N , with the result that the general equation (29) can be transformed into

$$\begin{aligned} & \frac{1}{3} N(N^2 - 1) (N + 2) (N + 3) x_m \\ &= \sum_{n=0}^{n=N} x_n [N(N - 1) \{(3N^2 + 3N + 2) \\ & \quad - 6(2N + 1)n + 10n^2\} \\ & \quad + 2m \{-3N(N - 1) (2N + 1) \\ & \quad \quad + 2(2N + 1) (8N - 3)n - 30Nn^2\} \\ & \quad + 10m^2 \{N(N - 1) - 6Nn + 6n^2\}], \end{aligned} \tag{36}$$

where x_m denotes the value of x at any time $t = m\delta$ and m is not necessarily an integer. It follows that

$$\begin{aligned} & \frac{\delta}{6} N(N^2 - 1) (N + 2) (N + 3) \left(\frac{dx}{dt}\right)_m \\ &= \sum_{n=0}^{n=N} x_n [\{-3N(N - 1) (2N + 1) + 2(2N + 1) \\ & \quad \times (8N - 3) n - 30Nn^2\} \\ & \quad + 10m \{N(N - 1) - 6Nn + 6n^2\}]. \end{aligned} \tag{37}$$

Explicit expressions for the variances, corresponding to (32) and (33), can obviously be written down.

For the purpose of evaluating q , however, the general expressions (36) and (37) are unnecessary, and it suffices to interpolate the values of x and dx/dt at any convenient point on the graph. With the joint object of minimizing the variance of the estimates and of obtaining the simplest formulae, it is advantageous to select the mid-point of the range, namely, $m = \frac{1}{2}N$, and, with this value of m , (36) and (37) reduce to

$$x_{\frac{1}{2}N} = \frac{3}{2(N^2 - 1) (N + 3)} \sum_{n=0}^{n=N} x_n \{- (N - 1) (N - 2) + 10Nn - 10n^2\}, \tag{38}$$

$$\left(\frac{dx}{dt}\right)_{\frac{1}{2}N} = \frac{6}{N(N + 1) (N + 2)} \sum_{n=0}^{n=N} x_n (2n - N). \tag{39}$$

For linear graduation at equal intervals (36) reduces to

$$x_m = \frac{2}{N(N + 1) (N + 2)} \sum_{n=0}^{n=N} x_n \{N(2N + 1 - 3n) + 3m(2n - N)\}, \tag{36a}$$

whence at the mid-point $m = \frac{1}{2}N$ we discover the obvious result $x_{\frac{1}{2}N} = \bar{x}$, and the interesting fact that the slope is given by a formula identical with (39). Thus linear and quadratic graduation lead to the same slope prediction at the mid-point, but not to the same prediction for the ordinate which may influence the evaluation of the correction factors in (24).

The result (30a) makes it somewhat tempting to reduce the variance of the estimated slope by clustering the observations at either end of the range, instead of keeping them equidistant in time. Such a procedure cannot be generally recommended, however, for the absence of points disclosing possible

curvature near the middle of the range is a high price to pay for the relatively minor reduction in variance that can usually be achieved in this way.

By applying (31) and (34) it is possible to write down the variances corresponding to (38) and (39), that for the slope being

$$\text{var} \left(\frac{dx}{dt} \right)_{\frac{1}{2}N} = \frac{36 \text{ var } p_s}{N^2(N+1)^2(N+2)^2 \delta^2} \sum_{n=0}^{n=N} \frac{(2n-N)^2}{p_{sn}^2}, \quad (40)$$

on the assumption that all the readings p_{sn} have a common variance, $\text{var } p_s$. For the approximate evaluation of (40) it will usually be sufficiently accurate to suppose that p_s suffers the exponential decay represented in terms of the rate of air change q by

$$p_{sn} = p_{s0} e^{-q\delta n},$$

so that

$$\text{var} \left(\frac{dx}{dt} \right)_{\frac{1}{2}N} = \frac{36 \text{ var } p_s \sum_{n=0}^{n=N} (2n-N)^2 e^{2q\delta n}}{N^2(N+1)^2(N+2)^2 p_{s0}^2 \delta^2}. \quad (41)$$

Thus there is an optimum value for the time interval δ between readings which minimizes the variance of the estimated slope $(dx/dt)_{\frac{1}{2}N}$. This optimum is defined by the transcendental equation

$$\sum_{n=0}^{n=N} (2n-N)^2 (q\delta n - 1) e^{2q\delta n} = 0, \quad (42)$$

which has a solution of the form

$$q\delta = f(N), \quad (43)$$

where $f(N)$ is an implicit function of N only. As might be anticipated, the optimum interval is inversely proportional to the rate of air change q . For any given value of N the function $f(N)$ may be evaluated numerically.

The case of $N=3$ is applicable to ventilation experiments in which it has been found convenient to take four measurements of p_s at regular intervals δ of time (see text-fig. 1), thus, in the case of quadratic graduation, leaving only one degree of freedom for least-square adjustment. Equations (38) and (39) now become explicitly

$$x_{\frac{1}{2}} = \frac{1}{18} \{9(x_1 + x_2) - (x_0 + x_3)\}, \quad (44)$$

$$\text{and} \quad \left(\frac{dx}{dt} \right)_{\frac{1}{2}} = \frac{1}{108} \{3(x_3 - x_0) + (x_2 - x_1)\}, \quad (45)$$

giving respectively the smoothed ordinate and smoothed slope of the $\log_e p_s$ graph at the mid-point of the range 3δ .

Since the four determinations of p_s involved are likely to have a common variance, $\text{var } p_s$, an application of (31) and (34) leads to the results

$$\text{var } x_{\frac{1}{2}} = \frac{\text{var } p_s}{256} \left(\frac{81}{p_{s1}^2} + \frac{81}{p_{s2}^2} + \frac{1}{p_{s0}^2} + \frac{1}{p_{s3}^2} \right), \quad (46)$$

$$\text{and} \quad \text{var} \left(\frac{dx}{dt} \right)_{\frac{1}{2}} = \frac{\text{var } p_s}{100\delta^2} \left(\frac{9}{p_{s3}^2} + \frac{9}{p_{s0}^2} + \frac{1}{p_{s2}^2} + \frac{1}{p_{s1}^2} \right). \quad (47)$$

In the present instance of $N=3$, equation (43) becomes explicitly

$$q\delta = 0.377, \quad (48)$$

the time unit being hours, so that if $q=4$ air changes per hour, the optimum value of δ is about $5\frac{1}{2}$ min. (cf. Experimental Technique, §4).

3.7. The human respiration of oxygen, carbon dioxide and water vapour

In order that equation (24) may be applied to occupied rooms, it is necessary to examine how the correction term involving v_s is affected by the respiration of the occupants, supposedly human. Apart from traces of organic matter, the only constituents of normal air that enter into active respiratory exchange with the human body are oxygen, carbon dioxide and water vapour, which may be designated respectively by the suffices $s=o$, c and w . Their rates of production per person, expressed in room volumes measured at N.T.P. (32° F. and 760 mm. of mercury column), will be denoted by $[v_s]$. Nitrogen ($s=n$) plays a purely passive role during steady breathing at normal atmospheric pressure, and may be dismissed, since $[v_n]$ is zero. The exchange of these three active substances will now be considered in some detail, with especial reference to experiments in which the selected tracer substance is CO_2 .

There would be no difficulty in quoting from the literature values of great diversity for the 'normal' rates of human production of CO_2 and water vapour, but in order to steer a rational course through this morass of data it is desirable either to select values that can be associated with a fairly definite state of muscular activity appropriate to the present problem, or else to indicate the functional dependence of the rates on the degree of activity. The following specifications of respiratory output are therefore linked first with the metabolic rate and thereby with the rate of performance of external work.

The rates of human absorption of oxygen and emission of carbon dioxide are connected by the relation

$$[v_c] + r[v_o] = 0,$$

where r , the respiratory quotient (namely, the ratio of the volumes of CO_2 expired to oxygen absorbed), depends to a large extent upon the relative proportions of carbohydrate and fat being metabolized by the subject, and normally lies between the limits of about 0.72 and 1.00. If r is interpreted as the non-protein respiratory quotient, the rate of oxygen consumption can be shown to be*

$$7.96 \times 10^{-3} (r + 3.52)^{-1} \quad (49)$$

* Thermochemical considerations determine the form of the expression as a function of r . The numerical coefficients accord with the data of Cathcart & Cuthbertson (1931).

N.T.P. cu.ft. per British Thermal Unit (B.T.U.) of total metabolic energy liberated.

A potent co-ordinating hypothesis concerning the metabolic rate is the widely accepted one that its basal level (or perhaps more precisely the part of it that is independent of external work) tends to be proportional to the surface area of the body (Du Bois, 1936; Brody, 1945), which according to the classical Du Bois formula (Du Bois & Du Bois, 1917) equals

$$S = 0.658m^{0.425}h^{0.725}, \quad (50)$$

where S is the area in square feet of a man of mass m lb. and height h ft.

Over a considerable range of activity, the metabolic rate proves to be a linear function of the rate of performance, W , of external work, the algebraic coefficient of W being inversely proportional to the mechanical efficiency of the process. This efficiency depends upon the nature of the task, varies considerably from person to person, and is influenced by training (Schneider, 1941). For ordinary domestic activity an efficiency of 20% seems appropriate and leads to the conclusion that in almost still air, at a temperature in the neighbourhood of 65° F., a normally clothed man, performing external work at the rate of W ft.lb./hr., liberates metabolic energy at the rate of*

$$20.5S + 6.42 \times 10^{-3}W \quad (51)$$

B.T.U./hr. The term independent of W has been adjusted to give a total rate of 400 B.T.U./hr. for a man having a standard body surface of 19.5 sq.ft., when seated quiescently awake in a chair, from which state as zero W is supposed to be reckoned.

From (49) the oxygen consumption corresponding to (51) is found to equal

$$-V[v_o] = (0.163S + 5.11 \times 10^{-5}W)(r + 3.52)^{-1} \quad (52)$$

N.T.P. cu.ft./hr., while the associated carbon dioxide output amounts to

$$V[v_c] = (0.163S + 5.11 \times 10^{-5}W)r(r + 3.52)^{-1} \quad (53)$$

N.T.P. cu.ft./hr.

On deducting the thermal equivalent of the external work from (51), the rate of output of heat (including both sensible and latent) is found to be

$$20.5S + 5.14 \times 10^{-3}W \quad (54)$$

B.T.U./hr.

* (1) E. H. Starling (1945) tabulates data which lead to the expression $(20.2S + 6.41 \times 10^{-3}W)$ in very close agreement with the expression (51). (2) D. Brunt (1947) gives an expression equivalent in our units to $(23.6S + 4.19 \times 10^{-3}W)$, which implies the rather high net efficiency of 30.7%; Brunt's 24% presumably refers to the gross efficiency at an external power-level of about 3.96×10^5 ft.lb./hr. (3) *The American Society of Heating and Ventilating Engineers Guide* (1947, p. 212) gives graphical data from which can be inferred the expression $(20.5S + 1.44 \times 10^{-3}W)$, implying the exceptionally low net efficiency of only 8.9%.

At air temperatures in the neighbourhood of 65° F., and in the resting state ($W=0$), about 80% (or 16.4S B.T.U./hr.) of this heat leaves the body by radiation and convection, the remaining 20%* (neglecting relatively trivial amounts carried off in urine and faeces) being accounted for by the latent heat of evaporation of water from the lungs and skin and by small changes in body temperature. If the combined convective and radiative loss remained constant in absolute magnitude (but not, of course, constant in percentage) during the performance of external work, and if there were no storage of heat in the body, the evaporative loss of heat would necessarily be $(4.10S + 5.14 \times 10^{-3}W)$ B.T.U./hr. Provided that the stage of profuse visible sweating has not been reached, this probably overestimates the influence of W , for under ordinary circumstances of working the convective loss can scarcely fail to be enhanced by body movements, and both radiation and convection will be augmented by any rise of skin temperature that may take place. In view of the general complexity of the phenomenon of perspiration (Kuno, 1934) and the apparent paucity of data specifying the variance to be expected in estimates of water loss associated with a given rate of working, a reasonable compromise may be made by halving the coefficient of W and writing the evaporative rate of loss of heat as

$$4.10S + 2.57 \times 10^{-3}W \quad (55)$$

B.T.U./hr. In the absence of thermal storage by the body this would imply a combined radiative and convective loss of $(16.4S + 2.57 \times 10^{-3}W)$ B.U.T./hr., which appears quite reasonable on independent grounds. Using the fact that the latent heat of vaporization of water at body temperature is 2.29 B.T.U./g., (55) becomes

$$1.80S + 1.13 \times 10^{-3}W \quad (56)$$

grams of water per hour.†

For the present application it is convenient if unconventional to treat the condensible water vapour

* For a man of area 19.5 sq.ft. this 20% amounts to 80 B.T.U./hr. and is equivalent to the evaporation of 35.0 g. of water per hour, which agrees well with the findings of Rubner (1896) and of Benedict & Root (1926). The figure of 24% (96 B.T.U./hr.) which is sometimes quoted (see Peters, 1935, Chapter VII) is probably on the high side, for it implies the evaporation of 42.0 g. of water per hour, which is beyond the limit of the usual range of from 30 to 40 g. per hour. Support for this contention can be found in Du Bois (1936, p. 65).

† For a 19.5 sq.ft. man, equation (56) becomes $(35.0 + 1.13 \times 10^{-3}W)$ and obviously lies comfortably between the extremes of $(37.7 + 2.28 \times 10^{-4}W)$, derivable from Starling's (1945) data and $(32.8 + 3.91 \times 10^{-3}W)$, derivable from the *A.S.H.V.E. Guide* (1947, p. 213), which respectively under-emphasize and over-emphasize the effect of W .

just as though it were a perfect gas and to express the water-vapour output as

$$V[v_w] = 7.89 \times 10^{-2}S + 4.95 \times 10^{-5}W \quad (57)$$

N.T.P. cu.ft./hr.

The information required for calculating the rate of human exchange of oxygen, carbon dioxide and water vapour is now assembled in expressions (52), (53) and (57). For the standard man of surface area 19.5 sq.ft., having a respiratory quotient of 0.85 and working at the rate of 2.5×10^4 ft.lb./hr.* (a value deemed to correspond to the performance of light domestic work while standing), it is found that the rates of production, in cubic feet per person per hour, are as given in the following table. The average woman will have rates of production equal to about 83 % of those tabulated:

Substance	Cu.ft./hr. at	
	N.T.P.	N.P. and 65° F.
Oxygen	-1.02	-1.09
Carbon dioxide	0.868	0.927
Water	2.78	2.96
Total volume	2.62	2.80

It is now possible to examine the magnitude of the correction terms in the expression (24) for q . For the sake of definiteness, consider one person in a room of volume $V = 1000$ cu.ft., in which the pressure remains constant while the temperature rises through the value 65° F. at the rate of 10° F./hr. If, at the given instant, the room air contains 4 % of CO₂ and the outer air 0.04 %.

$$(p_c - p'_c)^{-1} = 25.3, \quad v_c = 9.27 \times 10^{-4}, \quad \Sigma v_s = 2.80 \times 10^{-3},$$

whence $p'_c(p_c - p'_c)^{-1} = 1.01 \times 10^{-2},$

$$(v_c - p'_c \Sigma v_s)(p_c - p'_c)^{-1} = 2.34 \times 10^{-2},$$

and $\frac{d}{dt} \log_e T = 1.91 \times 10^{-2}.$

In this instance the correction for the presence of one person amounts to 0.0234 of an air change per hour, so that ten persons in the same room would falsify the result by between one-fifth and one-fourth of an air change per hour. This accords with the observation (§5E (ii)) that ten men in a room of volume about 1140 cu.ft. caused an error of about 0.23 of an air change in experiments in which the percentage of CO₂ fell from about 4 % to about 3 %. The effect varies directly as the number of persons per unit volume of room and almost inversely as the prevailing concentration of CO₂. If, in the present

* At this rate of working, equivalent to 32.1 B.T.U./hr., the metabolism is increased by 40 %, the total heat loss by 32 %, the combined radiative and convective loss by 20 %, and the evaporative loss by 80 %, above their respective resting levels.

example, $q = 1$, the total correction for atmospheric CO₂, human respiration, and temperature rise amounts to 0.0525 air changes per hour.

To make full use of equation (10) it would be of interest to know how the rates of production vary with the concentration of CO₂ in the inspired air. The information on this point is, however, somewhat scanty, and any regular variation that may be present appears to be largely masked by short-period storage effects which exhibit marked individuality even to the extent of changing sign. An analysis of an extensive early series of measurements made by Haldane & Priestley (1905) has indicated a slight but statistically insignificant increase of CO₂ output with concentration. Some such effect must inevitably result from the enhanced metabolism caused by the increased effort in breathing atmospheres containing several percent of CO₂, but the increase is too slight to be of practical consequence in a correction which is itself quite small.

Any useful discussion of the magnitude of the effects of losses by diffusion and absorption on the coefficients b'_i and $b_{i,r}$ in equation (10) must await the gathering of more factual knowledge than appears to be available at present.

It should not be assumed that other popular indicators, when used in the presence of human beings, are entirely exempt from the type of correction considered in relation to CO₂. Hydrogen, for example, which is an attractive choice on account of its ready detection in relatively low concentrations by means of the katharometer, may suffer some absorption by human beings, and its high diffusivity, although conducive to rapid dissemination, may also lead to appreciable non-ventilative loss. Carbon monoxide is notorious for its intense fixation by the blood, and human absorption could probably interfere appreciably with the low concentrations which alone would be hygienically permissible. The technique of measuring coal gas by the katharometer is spared some of this difficulty because the reading is determined more by the hydrogen content of the gas than by the carbon monoxide content.

3.8. The process of complete mixing

The complete mixing of the air content of the room which, as pointed out in §3.1, is essential to the present theory, can only be brought about quickly by the eddy-diffusive processes associated with a high degree of turbulence. True molecular diffusion is extremely slow in smoothing out differences of composition extending over distances comparable with the dimensions of a room, and is only of service in carrying to its molecular conclusion the process of homogenization already initiated by the entrainment of constituents in the small eddies and striae of a turbulent air stream.

Equation (1) is only valid as it stands if uniformity of ρ_s throughout the volume of the room is ensured by perfect mixing. If the mixing should be imperfect, the first term $V \frac{d\rho_s}{dt}$ of (1) must be replaced by $\frac{d\mu_s}{dt}$, where μ_s is the total mass of the constituent s contained in the room, and the ventilation terms in Q and Q' must be replaced by surface integrals taken over the room boundaries. The time variation of the density at any particular point is now a function of position which obeys the hydrodynamical continuity equation

$$\frac{d\rho_s}{dt} + \text{div}(\rho_s \mathbf{u} - \mathcal{D} \text{grad} \rho_s) = k_s, \quad (58)$$

where \mathbf{u} is the mean velocity vector (averaged over the turbulent fluctuations responsible for \mathcal{D}) of the air stream, \mathcal{D} is the effective diffusivity (or resultant of the eddy diffusivity and the relatively negligible molecular diffusivity) and k_s is the mass rate of generation per unit volume at the point in question. This equation may also be written in the form

$$\frac{d}{dt} \log_e \rho_s = - \frac{T}{P\rho_s} \text{div} \left(\frac{P}{T} \rho_s \mathbf{u} - \mathcal{D} \text{grad} \frac{P}{T} \rho_s \right) + \frac{RTk_s}{M_s P \rho_s} + \frac{d}{dt} \log_e \frac{T}{P}, \quad (59)$$

which should be compared with (24). There is, then, no unequivocal rate of air change capable of describing the variations of concentration at all points, but each point will have an associated 'equivalent' rate of air change,

$$q = \frac{T}{P\rho_s} \text{div} \left(\frac{P}{T} \rho_s \mathbf{u} - \mathcal{D} \text{grad} \frac{P}{T} \rho_s \right), \quad (60)$$

which is consequently a function of position and, as pointed out by Lidwell & Lovelock (1946), may not bear any simple relation to the mean rate of air change for the room as a whole. Equation (59) throws abundant light on the phenomena of subsidence and 'stratification' to be described in § 5D(v), and makes it clear that the rate of change of concentration at a point, being the result of the convection and diffusion of matter, depends largely upon the distribution of air currents and of turbulence in the room, and is therefore not necessarily controlled by the entry and exit phenomena taking place across the boundaries of the room.

The slowness of true diffusion may be illustrated by the fact that if, for example, CO_2 starts diffusing in still air from an imaginary plane behind which it is maintained at a fixed concentration p_0 , the concentration at a point situated at a distance z ft. in front of the plane does not rise to the value of even $\frac{1}{2}p_0$ until a time

$$t = 2.06z^2 \quad (61)$$

hours has elapsed.* The involvement of the square of the distance emphasizes the importance of preliminary turbulent dispersal; as z diminishes from 10 ft. to $\frac{1}{10}$ ft., t diminishes from about $8\frac{1}{2}$ days to 74 sec.

In contrast to this, turbulent diffusion is enormously faster. To form a rough estimate of the magnitude of the eddy diffusivity \mathcal{D} , the atmosphere of a room moderately stirred by a fan may perhaps be compared with the thermally stable and calm atmosphere, examined by Taylor (1915) over the Great Banks of Newfoundland during the *Scotia* expedition, which yielded values for the eddy diffusivity capable of being accommodated by the formula

$$\mathcal{D} = 153 \mathcal{W}^{2.71}, \quad (62)$$

\mathcal{D} being in (cm.)²/sec. and \mathcal{W} being the wind strength on the Beaufort scale. A light breeze of about 5 m.p.h. (Beaufort 2) would thus entail an eddy diffusivity of nearly 1000, which would suffice to hasten the transport of CO_2 by a factor of about 7000.

A gas like CO_2 , whose molecular weight exceeds that of air by 15 units (ratio 1.52), is very prone to subside and separate before diffusion can operate extensively, and it is necessary to ensure its thorough distribution by introducing it directly into the slipstream of a powerful fan (cf. § 5D(v)). With hydrogen, having a molecular weight less than that of air by 27 units (ratio 0.069), there is an even greater tendency for differential ascent. The fan should be kept running until the gas has been so minutely disseminated that diffusion can complete the process of molecular mixing. Thereafter, although no further gravitational separation need be feared in the already mixed air, the continuous advent of fresh air demands some stirring agency to preserve an equable mixture throughout the room during the course of a sampling experiment. It is useful to recollect that the rate of volume delivery of air by aerodynamically similar fans is proportional to the product of their speeds and the cube of their diameters.

Even when no fan is used, the atmosphere of a normally heated room is far from being merely quiescently diffusive, for it is not only ventilated by the aspiration of fresh air but is also pervaded by internal convective circulations caused by cold windows (katabatic currents), hot chimney-breasts or radiators (anabatic currents) and the like. Some 30-40% of the total metabolic heat emitted by the human body is dissipated by convection currents which rise with considerable penetrative power and

* The diffusivity of CO_2 in air is taken as 0.138 cm.²/sec. For hydrogen diffusing in air the coefficient of z^2 becomes 0.464, and the times require division by 4.43. The diffusivity in general varies roughly inversely as the square root of the molecular weight.

aid the dispersal of atmospheric constituents even when the subject is resting and holding his breath.

Human beings also contribute mechanically to the movement of room air (cf. §5D(iv)). Currents are caused by the bodily displacement (amounting to about 3 cu.ft.) of air as they move about, by the external bellows action of the chest, and by the forcible expulsion of air from the nostrils. Calculations have been made by Shaw (1907, p. 48) and revised by Duffon (1937) as to the temperature of the ambient air at which expired human breath just ceases to ascend. These authors respectively estimate the neutral temperature as 81 and 88° F., but it is doubtful if either value has much significance, since the expired air does not form a well-defined bubble whose buoyancy is calculable by elementary means but emerges as an eddy-bordered jet from which heat and matter are progressively lost by diffusion. In any case it is not possible to fix the neutral temperature within several degrees because the percentage of carbon dioxide in expired air differs considerably from person to person and, owing to the existence of a 'dead space' in the respiratory tract, actually varies with the depth of breathing.

3.9. *Density effects and the mechanism of influx and efflux*

As has been mentioned already in §3.3, the analysis hinging on the integration of equation (11) tacitly assumes that the ventilation rates Q and Q' of equation (1) or q of equation (9) are independent of the concentrations p_s of all the constituents of the air, and hence that the equations are linear. To examine this point it is necessary to see how the flows Q and Q' arise.

The mass flows represented by Q and Q' are essentially determined by the pressure differences existing between the interior and exterior of the room, and, in fact, represent an integrated result of numerous small transpirations taking place under a variety of pressure gradients acting through channels offering diverse degrees of throttling and frictional impediment. Broadly speaking, there are two types of flow according as the actuating pressure difference does work predominantly by overcoming frictional resistance in capillary channels (such as fine cracks) without engendering high velocities, or by generating (and possibly subsequently dissipating) kinetic energy in rapid efflux through ducts or patent orifices with relatively little friction. The volume rate of flow of air through porous materials and capillaries will be directly proportional to the pressure difference (in accordance with Darcy's law; and also Poiseuille's), whereas through orifices (and also approximately through most ducts) it will be proportional to the square root of the pressure difference (in accordance with Bernoulli's law). Incidentally,

it would appear that for most unsealed rooms orifice flow predominates, thereby accounting for the fact that ventilation rates are usually more closely related to the wind speed direct than to its square.

Among the various components contributing to the total pressure difference at any point across the boundaries of a room is one due to the different weights of the corresponding vertical air columns extending above that point inside and outside the room. This difference in terms of densities is equal to $gH(\rho - \rho')$ or to

$$\frac{gH}{R} \sum M_s \left(\frac{P}{T} p_s - \frac{P'}{T'} p'_s \right), \quad (63)$$

where g is the intensity of gravity and H the height of the room above the point in question, and hence depends linearly on the concentrations p_s of each constituent in the room. The general effect of this new term is to make q slightly dependent on p_s and thus to upset the strict linearity of equation (9) by introducing a higher order function of p_s . If q is taken to be approximately linear in p_s , which would be justified when the additional flow due to the term (63) forms only a small part of the flow otherwise existing, the addition amounts to a quadratic term in p_s , and the differential equation can still be solved although the solution proves to be rather cumbersome for practical application. Nevertheless, it appears that the density effects attainable with concentrations of several percent of a heavy gas like CO_2 could easily affect the motivating pressure differences by 10% and could therefore have a perceptible influence in the rate of air change, especially if the latter were low. Similar effects in the reverse sense would be experienced with hydrogen or any indicator having a radically different density from air.

It is, perhaps, worth mentioning in this context that any fans used for air-mixing should not be sufficiently powerful, or should not be so ill-placed, as to induce pressure differences on the walls (especially in the neighbourhood of any apertures for air flow) comparable with those responsible for the ventilation flow that is being measured (cf. §5D(v)). The pressure differences caused by the impingement of the slip-streams of aerodynamically similar fans may be expected to be proportional to the product of the air density and the square of the product of their speeds and diameters.

3.10. *The interaction of coupled rooms*

It is implicit in the ventilation process discussed in §§3.1–3.4 that the air contained in the room should be progressively evacuated to an extensive external atmosphere in which the concentration p'_s of any constituent s , although possibly variable, is quite unaffected by the amount of s discharged. This is no longer true when a part of the ventilation of

a room passes through an adjoining room, cupboard or hall which is in turn imperfectly ventilated to the outer air, for then the concentration in either chamber can influence the rate of change in the other, the two forming an interacting coupled system. In this case, even while the degree of ventilation, as measured by the volume flow of air, remains constant, the concentration-time curve for each room involves two distinct exponential subsidence terms and cannot be rectified by taking logarithms. It is thus possible for incompletely sealed cupboards of appreciable size to cause some distortion of the normal exponential decay curves and hence to falsify somewhat the ventilation rates derived therefrom.

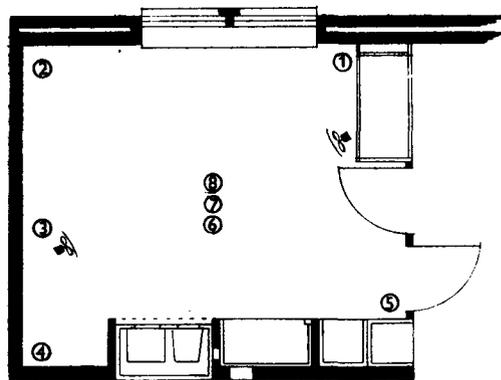
Taking a wider view of the problem, it will be seen that in general the rooms and passages of almost any building, together with the external atmosphere, form a multiply-coupled system for which the simultaneous differential equations can be written down after the fashion of equation (1). Just as in the more subtle example of coupling already mentioned in §3.3, the time variation of the concentrations in individual compartments of the coupled system can only be defined independently through the medium of a differential equation whose order is determined by the number of compartments that are significantly coupled. For a pair of rooms (denoted by the suffices 1 and 2) ventilating mutually and to the atmosphere (denoted by the suffix 0), the equations analogous to (1) would (dropping the constituent suffix s for the present) assume the form

$$\left. \begin{aligned} V_1 \frac{d\rho_1}{dt} &= K_1 + Q_{01}\rho_0 + Q_{21}\rho_2 - (Q_{10} + Q_{12})\rho_1, \\ V_2 \frac{d\rho_2}{dt} &= K_2 + Q_{02}\rho_0 + Q_{12}\rho_1 - (Q_{20} + Q_{21})\rho_2, \end{aligned} \right\} \quad (64)$$

where $Q_{\alpha\beta}$ in general represents the volume rate (measured at temperature T_α and pressure P_α) at which air passes from space α into space β . There would, of course, be a similar pair of equations for each constituent s . The formidable array of mutual ventilation coefficients $Q_{\alpha\beta}$ required for an adequate description of the process seems somewhat complicated to be likely to lead to results of practical value in ventilation measurement, at least in the present stage of development of the subject; but it is worth recollecting that a similar array of transmission coefficients is encountered in the closely cognate problem of heat exchange between rooms. Indeed, since the flow of air that constitutes ventilation inevitably transports heat as well as matter, it is clear that the differential equations of ventilative transfer, such as (64), form an essential part of a more comprehensive theory of combined heating and ventilation, which will surely require deeper investigation in the future.

4. EXPERIMENTAL TECHNIQUE

In general, an initial concentration of about 2–4% CO_2 was produced in a room, but with air changes greater than about 5 per hour, the initial concentration corresponded to 4–6% CO_2 . In the latter case, however, owing to rapid loss of gas, the concentration was lower when the experiment started (cf. Tables 5 and 6). The gas was liberated from a cylinder into large Douglas bags of 500 or 1000 l. capacity, allowed to stand for at least $\frac{1}{2}$ hr. to reach room temperature and then passed into a room, either through a wide corrugated rubber tube passing through a hole in the door, or liberated directly into the room. In either case the emergent stream was passed in front of an electric fan. For routine work in rooms of 1000–1500 cu.ft. volume, a total of two fans (10 in. diameter, 620 r.p.m.) were in operation during the entry of the gas and throughout the whole of an experiment, and this produced a minimum air movement of about 40 ft./min. at any point in the room. During the experiment these fans were on the floor of the room, pointing upwards towards the opposite corner as shown in Text-fig. 2. While the gas



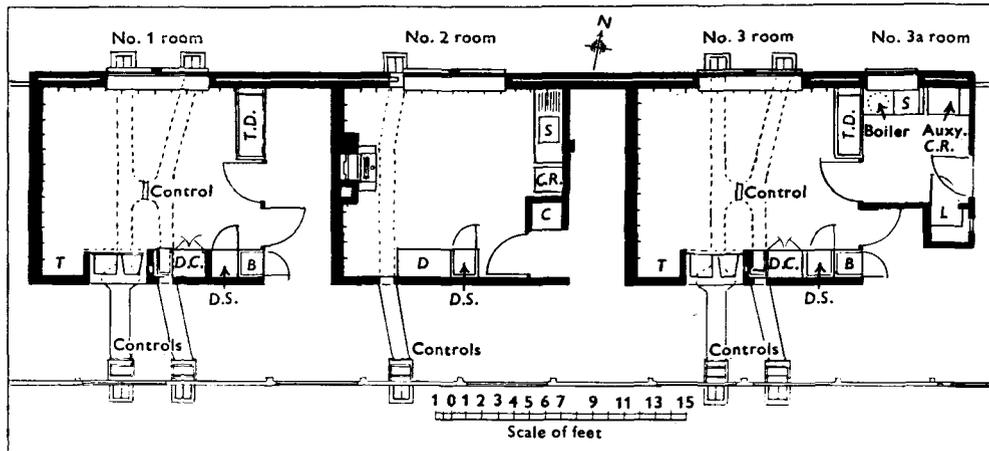
Text-fig. 2. Experimental room showing routine position of fans and sampling points used in the estimation of routine mixing error. Numbered positions where samples were taken. Samples 1, 2, 3 and 5 taken at mid-vertical level of the wall. Sample 4, in corner recess 6 in. from the floor. Samples 6, 7 and 8 vertical plane centre of room: no. 6 at 3 in. from floor; no. 7 at centre point; no. 8 at 1 in. from ceiling.

was being passed into the room, however, the fan nearer the door was placed so that the gas coming in through the corrugated rubber tubing could be passed over it. In some experiments done in a room containing a fire or open window, additional fans were arranged to break up the main air stream without themselves modifying air movement in or out of the openings. Unless otherwise stated, fans

were used as a mixing agent in all experiments described in this paper. Air sampling was commenced 2–3 min. after completion of entry of the gas. As a rule, air samples were taken at intervals of 5 min., and timed by a stop-watch, but in some cases with air change over 5 per hour, samples taken at 2½ min. were more suitable. Acidulated and evacuated Haldane gas sampling tubes of 75 c.c. volume were used, and all tubes were tested for leak prior to use. When samples were taken by an observer in the room, the breath was held for 15 sec. prior to sampling, and the tube held at arm's length at the required point in the room. This minimized contamination by expired air. For routine experiments in closed rooms it was usual to take samples in the middle of the room above head level. Estimations of concentration of carbon dioxide in the air samples were made with a standard 10 c.c. Haldane gas analysis apparatus (Haldane & Graham, 1935).

The method of 'least squares' eliminates individual idiosyncrasies in drawing straight lines and has been adopted as it provides a simple standard method. A discussion on the matter is given in Theoretical Analysis, §3.6. The statistical methods used in the analysis of the experimental data are standard methods and the 't' and the 'F' test for significance have been used.

Most of the work to be described was done in experimental kitchen living-rooms at the Ministry of Works Field Test Unit at the Thatched Barn, Barnet, Herts. The rooms were built within, and shared one external wall with, a Ministry of Works Standard Hut. Of these, two rooms (room 1 and room 3 of Text-fig. 3) were made as nearly alike as modern constructional methods allow, except that room 3 communicated with an annexe by a door. These rooms have distempered walls and ceilings. The floors are covered with linoleum marked into



Text-fig. 3. Plan of experimental kitchen living-rooms. *B*, broom cupboard; *C*, cupboard; *D.C.* drying cupboard; *D.S.* dry stores; *T.D.* through dresser; *D*, dresser; *T*, table; *C.R.* cooker; *S*, sink; *L*, larder.

One air change per hour denotes the removal of a volume of room air equal to the volume of the room, in 1 hr., and its replacement by an equivalent volume of outside air. The total volume of air removed per hour is thus the product of room volume and air change.

Air change (q) has been estimated from the slope of the $\log_e(\text{CO}_2 \text{ concentration})$ versus time curve. The line is assumed straight and the slope found by means of the method of 'least squares'. Except where mentioned, no corrections have been used. The assumption of a simple linear relationship is equivalent to using only the term

$$q = -\frac{d}{dt} \log_e p_s$$

of Theoretical Analysis equation (24).

squares allowing the instruments to be placed in more or less identical positions in the two rooms. Both rooms have the same type of fireplace and flue, but there is no other ventilating opening. The same type of furniture is present in both rooms and kept in similar positions. Unless otherwise stated, during all experiments quoted below, the opening into the fireplace was sealed off and doors, windows and cupboards closed. The thermal characteristics of the rooms were measured by a silvered thermometer and globe thermometer placed at corresponding positions in each room. Readings of wind velocity at the outside wall were taken by small Sheppard-type cup-anemometers (Sheppard, 1939) which gave a measure of the horizontal component. The rooms are in telephonic communication, so that paired experiments could be synchronized.

External sampling was done where necessary using the apparatus in Pl. 1, figs. 1, 2. Leads of glass tubing and rubber pressure tubing pass from selected sampling points within a room to evacuated sampling tubes connected as shown, and to a 5 l. water bottle acting as a syphon. The volume of the connecting tubes is small compared with that of the enclosure, so a representative air sample is ensured by flushing the tubes out by drawing air from the enclosure to the syphon for a short period immediately before opening the connector to the sampling tube. To eliminate the small time error present when using three sampling tubes (about 2-3 sec.), the order is reversed at each sampling. The apparatus permits testing for leaks at all points in the circuit. Prior to an experiment, all parts of the apparatus are acidulated with 5% sulphuric acid. Pl. 1, fig. 1 shows the apparatus set up for a comparison of simultaneous internal and external sampling (cf. §5D(v)).

As an example of the use of the above technique for the measurement of air change, the following experiment is quoted:

Simultaneous duplicate experiment—natural air change

Date. 14 July 1947.

Meteorological conditions. Hot, overcast, calm.

Beaufort scale. 0-1 (equivalent to 0-3 m.p.h.).

Method. The experiments were done in rooms 1 and 3, in each of which was one observer. Two routine fans were set in operation, one of them being close to the door. The windows, cupboards and all openings in rooms 1 and 3 were closed and approximately 700 l. of CO₂, collected and stored in Douglas bags, were passed into each room over the fan placed nearest the door. As soon as all the gas had been liberated (about 10 min.) the door fan was directed towards the corner of the room (cf. Text-fig. 2). After a further 2 min. of mixing, sampling was commenced from the centre of the room above the head, after the breath had been held for 15 sec. Simultaneous samples were taken in each room at 0, 5, 10, 15 min. The fans continued in action during the whole of the experimental period. At the conclusion of the experiment the samples were analysed in the standard 10 c.c. Haldane gas analysis apparatus, and gave the following results:

Time (min.)	CO ₂ (%)	
	Room 1	Room 3
0	2.22	2.74
5	2.02	2.53
10	1.93	2.32
15	1.88	2.22

The general formula for obtaining the air change from the observations of tracer gas concentration is

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given in the Theoretical Analysis equation (39). In the case of four samples taken at 5 min. intervals, this simplifies to equation (45) which may be rewritten as

$$\text{Air change} = 1.2 \{3(\log_e p_{s0} - \log_e p_{s3}) + (\log_e p_{s1} - \log_e p_{s2})\}.$$

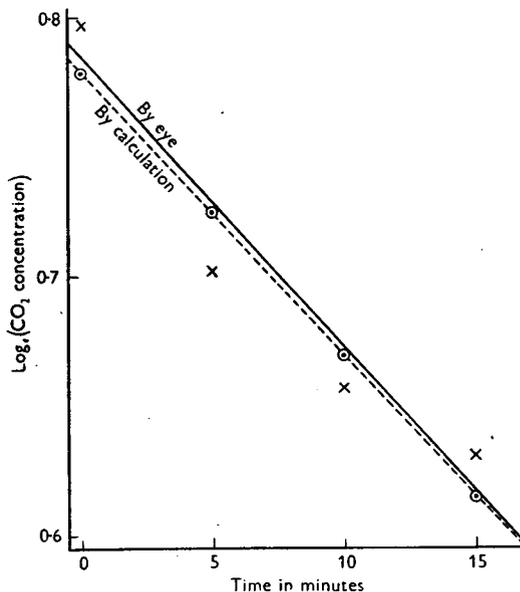
For room 1 we have

Time	CO ₂ concentration	log _e (CO ₂ concentration)
t ₀ 0 min.	p _{s0} = 2.22 %	0.7975
t ₁ 5 min.	p _{s1} = 2.02 %	0.7031
t ₂ 10 min.	p _{s2} = 1.93 %	0.6575
t ₃ 15 min.	p _{s3} = 1.88 %	0.6313

Hence the air change

$$= 1.2 \{3(0.7975 - 0.6313) + (0.7031 - 0.6575)\} = 0.65 \text{ per hr.}$$

The standard deviation (s.d.) calculated from equation (47) is 0.13 per hr. Similarly for room 3, the air change may be calculated to be 0.86 per hr.



Text-fig. 4. Decay of log_e (CO₂ concentration) with time.

Alternatively, the air change may be estimated graphically by plotting log_e (CO₂ concentration) against time, fitting a straight line to the points by eye, and calculating the air change from the slope of the line obtained. The air change per hour is the change in the logarithm of the concentration of CO₂ divided by the time in hours. For optimum sensitivity, the scales should be so chosen that the

slope of the line obtained is about 45° (i.e. 30° to 60°). The data from room 1 has been plotted in this way in Text-fig. 4, and a straight line fitted by eye to the four points. If the experimental points are not too scattered, the air change obtained in this way will agree fairly well with that derived by the method of least squares. Otherwise if the scattering is marked the error of any method is high. The method of least squares has the advantage of speed and simplicity when the time intervals between samples are equal and the technique standardized.

5. EXPERIMENTS AND RESULTS

A. *Physical characteristics of experimental rooms*

Since it was intended to use the two rooms 1 and 3 as controls one for the other, measurements were made of their principal physical attributes and critically compared.

Room volume. The principal measurements were made to the nearest quarter inch, and results are: room 1 = 1141 cu.ft.; room 3 = 1145 cu.ft. This difference appears to be well within errors of measurement, since the principal measurements are roughly 100 ins., 150 ins., and measurements are liable to an error of up to $\pm \frac{1}{8}$ in. Using accepted methods of calculating limits of error, these figures give as estimate of error $\pm 0.29\%$. Such an error in 1140 cu.ft. corresponds to a difference of about 3.3 cu.ft. Compared with this the observed difference of 4 cu.ft. is not great. It should be borne in mind, however, that the volume of the room is indeterminate to the extent of about 5% owing to the cupboard space.

Room temperature. A series of readings were taken at three fixed times a day, by the same observer, of a globe thermometer and silvered thermometer in the same position in both rooms. By comparisons made with a reading telescope outside the window, it had been shown that the error in reading due to the entry of an observer and proximity to the thermometers was negligible, and that due to the short interval of time between the readings in the two rooms was again negligible. The measurements were taken during 18 days (September–October 1947) when there was no corridor heating in the hut, and during 17 days (October–November 1947) when corridor heating was used. The results are summarized in Table 1.

External wind measurements. The wind conditions to which the two rooms are exposed were compared by means of simultaneous wind-velocity measurements at corresponding points outside the two rooms. Measurements were taken at three vertical heights, 2 ft. from the external wall, using nine Sheppard-type cup-anemometers. Wind speed was measured on nine different days spread over a period of some weeks at fixed times, once in the morning and once

in the afternoon. Since calibration errors were suspected, differences between the instruments were reduced by redistributing the various instruments at random among the nine positions for each set of observations. From the figures obtained, corresponding positions outside the two rooms were compared and the mean differences subjected to a 't' test for significance. The estimate of the s.d. of the difference contains a term due to the variability of the instruments among themselves, but a larger experiment, of which the above was part, failed to detect a significant variability. The results are given in Table 2.

Results. The volume difference between the rooms is negligible, and for practical purposes the rooms can be regarded as identical. In view of the restricted variety of weather conditions under which the temperature and wind measurements were made it is not possible to draw any firm conclusions concerning the existence or otherwise of differences between the rooms under particular conditions. However, the results indicate that:

(1) When the building is unheated, the mean difference in air temperature and globe thermometer readings is negligible and not statistically significant.

(2) When the corridor is heated by means of hot-water pipes from an outside boiler, there is a small but statistically significant mean difference between the rooms, that near the boiler (room 1) having a mean temperature about 1° F. higher.

(3) There is no significant difference between the mean wind velocity outside the rooms at any of the three points at which measurement was made.

B. *Errors of carbon dioxide estimation by the Haldane method*

To estimate this error, thirty-seven duplicate estimations from the same sampling tubes done by the same technician over a period of some months were examined. The results are given below:

Nine duplicates on external air

Mean difference between sample and duplicate = 0.008 %.

s.d. single difference = 0.022 %.

For 8 degrees of freedom $t = 1.00$ (approx.). Not sig.

28 duplicates on CO₂ concentrations up to 5.4 %

Mean difference between sample and duplicate = 0.010 %.

s.d. single difference = 0.052 %.

For 27 degrees of freedom $t = 1.00$ (approx.). Not sig.

It appears that there is no significant difference between a first estimation and its duplicate. Since the s.d. of a difference is $\sqrt{2}$ times that of the individuals whose difference is being estimated, the s.d.

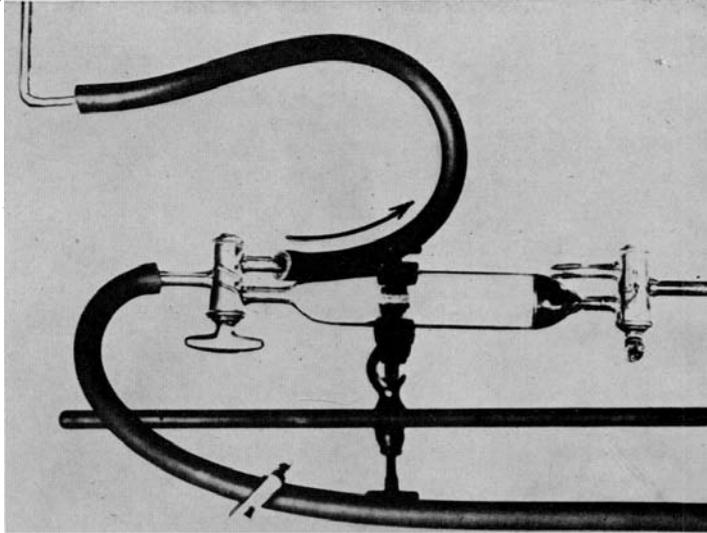


Fig. 2. External sampling tube showing tube for aspiration of dead space air before opening tap to position shown.

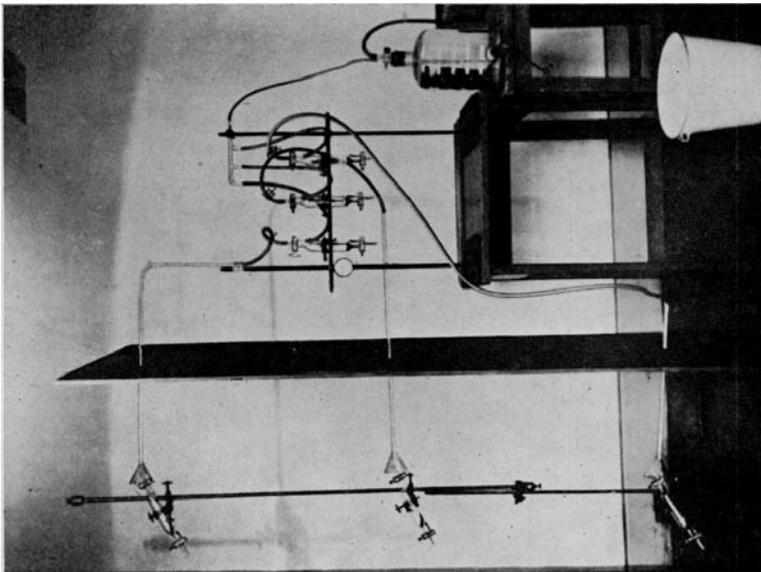


Fig. 1. Apparatus for internal or external sampling of room air at three levels.

Table 1. *Temperature difference (rooms 1-3)*

Instrument ...	Unheated corridor. Observations on 18 days					
	Silvered thermometer			Globe thermometer		
	10.15 a.m.	12.15 p.m.	3.30 p.m.	10.15 a.m.	12.15 p.m.	3.30 p.m.
Time of observation ...						
Mean outside temp. dry bulb (° F.)	50.1	55.3	58.0	50.1	55.3	58.0
Mean (° F.) (rooms 1 and 3)	51.37 51.46	52.71 52.89	54.33 54.18	51.81 51.72	53.09 53.11	54.71 54.33
Mean difference (° F.) (rooms 1-3)	-0.09	-0.18	+0.15	+0.09	-0.02	+0.38
s.d. difference (° F.) (rooms 1 and 3)	0.40	0.50	0.49	0.55	0.82	0.88
<i>t</i>	0.93	1.55	1.31	0.69	0.086	1.85
<i>P</i>	0.3 < <i>P</i> < 0.4 Not sig.	0.1 < <i>P</i> < 0.2 Not sig.	0.2 < <i>P</i> < 0.3 Not sig.	<i>P</i> = 0.5 Not sig.	<i>P</i> > 0.9 Not sig.	0.05 < <i>P</i> < 0.1 Not sig.
Combined estimates (3 times daily) (53 D.F.)	<i>t</i> = 0.63 0.5 < <i>P</i> < 0.6; Not sig.			<i>t</i> = 1.46 0.1 < <i>P</i> < 0.2; Not sig.		

Instrument ...	Heated corridor. Observations on 17 days					
	Silvered thermometer			Globe thermometer		
	10.15 a.m.	12.15 p.m.	3.30 p.m.	10.15 a.m.	12.15 p.m.	3.30 p.m.
Time of observation ...						
Mean outside temp. dry bulb (° F.)	45.6	47.9	48.0	45.6	47.6	48.0
Mean (° F.) (rooms 1 and 3)	52.13 51.57	52.26 53.31	54.13 53.21	52.82 51.99	53.81 52.69	54.55 53.55
Mean difference (° F.) (rooms 1-3)	+0.56	-1.05	+0.92	+0.83	+1.12	+1.00
s.d. difference (° F.)	0.94	0.51	0.86	0.68	0.51	0.80
<i>t</i>	2.477	8.01	4.38	5.01	9.11	5.15
<i>P</i>	0.02 < <i>P</i> < 0.05 Sig.	<i>P</i> < 0.01 Sig.				

In all cases a positive sign indicates that the mean value for room 1 was higher than for room 3, and a negative sign vice versa.

Table 2. *Wind-speed difference (rooms 1-3). Observations on 9 days*

Position of instrument ...	Window-sill level		Window-top level		1 ft. above roof	
	Morning	Afternoon	Morning	Afternoon	Morning	Afternoon
	11.20 a.m.- 12 noon	2.20- 3.00 p.m.				
Time of observation ...						
Mean (ft. sec.) (rooms 1 and 3)	4.87 5.21	5.02 4.99	5.35 5.43	5.32 5.54	6.55 6.57	6.26 6.05
Mean difference ft./sec., (rooms 1-3)	-0.34	+0.03	-0.08	-0.22	-0.02	+0.21
s.d. difference ft./sec. (8 D.F.)	0.67	0.85	0.62	0.68	0.80	0.30
<i>t</i>	1.65	0.12	0.38	0.99	0.08	2.10
<i>P</i>	0.1 < <i>P</i> < 0.2 Not sig.	<i>P</i> > 0.9 Not sig.	0.7 < <i>P</i> < 0.8 Not sig.	0.3 < <i>P</i> < 0.4 Not sig.	<i>P</i> > 0.9 Not sig.	0.05 < <i>P</i> < 0.1 Not sig.
Combined estimate morning and afternoon (17 D.F.)	<i>t</i> = 0.85 0.4 < <i>P</i> < 0.5; Not sig.		<i>t</i> = 1.01 0.3 < <i>P</i> < 0.4; Not sig.		<i>t</i> = 0.67 0.5 < <i>P</i> < 0.6; Not sig.	

The s.d. of difference was obtained from paired values in all cases. In all cases a positive sign indicates that the mean value for room 1 was higher than for room 3, and a negative sign vice versa.

of a single estimation of the higher concentrations is about 0.036 % CO₂.

We have evidence that the errors involved in sampling the same air by duplicate tubes, in duplicate sampling from the same tube, and in introduction of an air sample into the Haldane apparatus are, *per se*, negligible, so that the error given above arises

within the apparatus itself, and is due mainly to difficulty in reading small volume differences.

Evidence is also available that an element of bias is present in the routine estimation of duplicates, since the technician is aware of his first results. The real error in measurement of CO₂ may hence be greater than that given above.

C. Error of air-change estimation using simultaneous duplicate experiments

It has been known since the time of Pettenkofer that the air change in a closed room is dependent upon external weather conditions. Preliminary work in a small cubicle situated in a large laboratory under conditions of small changes in air temperature and movement, show that a small variation was present when determinations were done on six successive mornings. The results were: mean = 0.36 air changes per hour; s.d. = 0.0376. Ordinary rooms exposed to outside weather conditions show a much wider variation. Thus, in a series of determinations in room 1 done for 12 days over a period of some months, the results were: mean = 0.87; s.d. = 0.48 air change per hour. In order to ascertain the magnitude of variation during a day, a series of air changes were done eight times a day in room 1, the interval between the estimations being $\frac{3}{4}$ -1 hr. In

2nd day: flues open (8 estimations)

Mean air change = 2.21 per hour.
 Variance of daily air change (7 D.F.) = 0.2611; s.d. = 0.511.
 'Internal variance' (16 D.F.) = 0.0227.
 $F = 11.50$; Sig. (1% level).

3rd day: flues closed (7 estimations)

Mean air change = 1.00 air changes per hour.
 Variance of daily air change (6 D.F.) = 0.0311; s.d. = 0.177.
 'Internal variance' (14 D.F.) = 0.0076.
 $F = 4.01$; Sig. (5% level).

Table 3. Variation in air change during a day

1st day, fireplace open		2nd day, fireplace open		3rd day, fireplace sealed	
Air change at hourly intervals	Wind speed (ft./sec.)	Air change at hourly intervals	Wind speed (ft./sec.)	Air change at $\frac{3}{4}$ -hourly intervals	Wind speed (ft./sec.)
1.99	5.6	2.68	10.1	(2.58)	(4.2)
1.65	4.3	2.81	12.5	1.23	5.7
1.16	4.4	2.51	13.1	1.15	6.7
1.50	4.2	1.90	9.3	0.87	6.3
1.66	5.0	1.86	10.8	0.77	6.2
1.85	5.9	2.64	11.6	0.84	6.2
1.61	4.3	1.79	9.9	1.12	6.6
2.17	5.8	1.45	6.4	1.04	6.4

the first two days the flue was open and on the third day it was sealed. The variance of the air change has been compared with what we have called the 'internal variance'. This is an estimate of variance based on deviations of the observed points from the 'best straight line' (Fisher, 1946, §26), and corresponds to the expression for variance given in equation (33) of the Theoretical Analysis. The results, together with the corresponding wind speeds at roof level, are given in Table 3. The analysis of the data is given below:

1st day: flues open (8 estimations)

Mean air change = 1.70 per hour.
 Variance of daily air change (7 D.F.) = 0.0959; s.d. = 0.310.
 'Internal variance' (16 D.F.) = 0.0342.
 $F = 2.80$; Sig. (5% level).

From these results it is clear that the variability of air change over short periods of time is significantly greater than the error of estimation as indicated by the 'internal variance'. Table 3 shows that with periods of time as short as $\frac{3}{4}$ hr. a considerable change may occur, probably due to variations in external wind speed or direction. It is of interest that in spite of the highly complex relationship of the wind flow to ventilation (Carne, 1946) analysis of these figures for air change and roof wind velocity gave a correlation coefficient $r = 0.77$ (Sig. 0.1%, 14 D.F.) for a room with open flues. In the room with flues closed the scatter diagram between air change and wind speed consists of 6 points close together and 2 comparatively isolated points. For this reason it was not deemed advisable to compute the correlation coefficient. In the analysis above (3rd day) one of the two isolated points (see Table 3) has been omitted, since the air change appears suspiciously high. It becomes obvious that time itself is an important variable in air-change measurement, and in experiments done in the same room under different

internal conditions, the difference in air change found may be partly or completely masked by changes due to uncontrollable time variables. Because of such considerations it was decided to carry out simultaneous duplicate experiments in rooms 1 and 3 to estimate the error of air-change determinations.

A series of twenty-six experiments was made, where one observer was present in each room and internal conditions in the rooms were as alike as possible. The physical condition of these rooms has already been given. For the purpose of analysis the series was divided into three groups: (1) natural ventilation (15 experiments), (2) fires lit (7 experiments), (3) windows open (4 experiments). The data so obtained were subjected to analysis of variance, and the differences between the rooms compared with the 'internal variance' (Table 7).

(1) *Natural ventilation*

The experiments on natural ventilation are summarized in Table 4. In each case, except in Exps. 1 and 2, the rate of ventilation was measured over a 15 min. period and simultaneous air samples taken from the centre of the room at 0, 5, 10 and 15 min. The first eight experiments were done with the fireplaces sealed, so that the rooms were effectively flueless, and in the subsequent seven experiments the fireplaces were open. Since the variance of the former was significantly lower than the latter, these two groups of experiments have been analysed separately. The results are summarized in Table 7.

Table 4. *Duplicate air change in rooms 1 and 3. Natural ventilation*

Exp. no.	Time (min.)	Room 1		Room 3	
		% CO ₂	Air change and 'internal variance'	% CO ₂	
1	0	4.56	0.004733	1.20	4.55
	20	3.00			3.04
	40	2.13			2.08
	60	1.62			1.36
2	0	5.38	0.000325	0.72	4.45
	10	4.80			4.06
	20	4.24			3.49
	30	3.83			3.13
3	0	1.84	0.001270	0.64	3.14
	5	1.73			2.97
	10	1.63			2.75
4	0	2.02	0.007869	0.72	2.62
	5	1.91*			2.52
	10	1.86			2.28
	15	1.71			2.22
5	0	1.92	0.000250	0.77	2.84
	5	1.82			2.63
	10	1.71			2.49
	15	1.61			2.34

6	0	2.01	0.012473	0.84	1.87
	5	1.86			1.77
	10	1.83			1.67
	15	1.68			1.51
7	0	2.22	0.017367	0.86	2.74
	5	2.02			2.53
	10	1.93			2.32
	15	1.88			2.22
8	0	2.85	0.002772	0.78	1.93
	5	2.64			1.72
	10	2.52			1.67
	15	2.33			1.57
9	0	2.53	0.030997	1.17	3.79
	5	2.22			3.39†
	10	2.18			3.10
	15	2.01			2.83
10	0	2.73	0.019157	1.50	2.79
	5	2.58			2.52
	10	2.23			2.18
	15	2.05			1.93
11	0	4.51	0.003665	1.18	3.25
	5	4.21			3.02
	10	3.91			2.74
	15	3.54			2.42
12	0	2.45	0.001279	1.81	2.85
	5	2.08			2.45
	10	1.73			2.22
	15	1.47			1.78
13	0	2.89	0.024044	1.56	2.43
	5	2.39			2.02
	10	2.03			1.83
	15	1.82			1.63
14	0	3.56	0.009973	2.14	2.89
	5	3.16			2.43
	10	2.63			1.93
	15	2.28			1.72
15	0	2.99	0.016945	2.01	2.90
	5	2.51			2.34
	10	2.03			1.99
	15	1.78			1.75

* 6 min. † 5½ min.

(2) *Ventilation with fires lit*

The results of these experiments are summarized in Table 5. The first four experiments were of 15 min. duration, and the following three were of 7½ min. duration. Except where otherwise specified the air samples were taken at equal time intervals. Since the variance of the air change in the shorter experiments proved significantly higher than in the longer experiments the two groups have been analysed separately.

(3) *Ventilation with open windows*

All experiments were of 15 min. duration, and four samples were taken at equal intervals. The results are summarized in Table 6. Two experiments with open windows at rates of air change over 7 had variances so high that they have not been included in these results.

Table 5. Duplicate air change in rooms 1 and 3. With fires lit (windows shut)

Exp. no.	Time (min.)	Room 1		Room 3		
		% CO ₂	Air change and 'internal variance'	% CO ₂	Air change and 'internal variance'	
1	0	2.79	4.04	0.012753	0.134553	1.01
	5	2.02				0.71
	10	1.47				0.56
	15	1.01				0.35
2	0	1.91	3.61	0.033251	0.016100	1.01
	5	1.39				0.72
	10	1.01				0.55
	15	0.76				0.40
3	0	2.57	4.31	0.022750	0.019531	2.37
	5	1.77				1.70
	10	1.20				1.26
	15	0.88				0.86
4	0	2.80	3.58	0.024290	0.002935	2.45
	5	2.06				1.77†
	10	1.51*				1.38
	15	1.12				1.01
5	0	2.57	4.80	0.128865	0.140315	2.22
	2½	2.22				1.83
	5	1.71				1.47
	7½	1.44				1.10
6	0	2.56	3.66	0.019151	0.317441	1.88
	2½	2.24				1.62
	5	1.92				1.23
	7½	1.62				1.12
7	0	2.23	3.85	0.010299	0.084756	2.07
	2½	1.87				1.76
	5	1.62				1.47
	7½	1.37				1.16

* 11 min.

† 5½ min.

Table 6. Duplicate air change in rooms 1 and 3. With windows open (no fires)

Exp. no.	Time (min.)	Room 1		Room 3		
		% CO ₂	Air change and 'internal variance'	% CO ₂	Air change and 'internal variance'	
1	0	2.75	3.32	0.026226	0.022943	2.62
	5	2.04				1.82
	10	1.51				1.33
	15	1.21				1.00
2	0	3.51	4.70	0.159698	0.007581	3.55
	5	2.27				2.28
	10	1.43				1.52
	15	1.11				1.02
3	0	2.03	7.07	0.043725	0.210705	2.41
	5	1.16				1.17
	10	0.60				0.66
	15	0.35				0.41
4	0	4.39	4.73	0.278990	0.170545	4.87
	5	2.64				2.78
	10	1.72				2.02
	15	1.36				1.36

It appears to us that at rates of air change as high as 10 per hour, the CO₂ tracer method is not at all suitable, at least if the high rate of ventilation is due to open windows. Whether, indeed, any method is reliable, and whether the underlying assumptions in the concept of air change are valid under these extreme conditions, is open to question. Further data on this subject is given below (cf. § 5D (iii)).

In four of the five groups of experiments analysed (Table 7) the mean air change in room 3 is greater than in room 1, and in two groups the difference is significant. Such a difference is not likely to be due to air-temperature differences between the rooms or to a difference in external meteorological conditions. Room 3 opens on to an annexe which is absent in room 1, and it is possible that the 'effective crack area' in room 3 may be greater. The consistent difference in the 7½ min. experiments with fires lit may be due to the fact that the experiments were done over a period of only 4 days when weather conditions and wind direction did not vary greatly. The large difference found in these experiments clearly indicates that under some conditions of experiment, the difference between an experimental room and its control may be considerable. This emphasizes the advantage, where possible, of using balanced experiments; that is to say, experiments in which all controllable factors except those of immediate interest are arranged to occur equally in both sides of any comparison, so that linear effects are eliminated. The reliability of the control depends on the nature of the ventilation and is lower with those types of ventilation which give high air changes. The experimental results show that the 'internal variance' increases markedly as the air change increases. This indicates that the effect of the higher initial concentration used at high air change is more than compensated by the term e^{2abn} in equation (41). Shorter experiments were introduced in an attempt to minimize the error of experiment at high air changes, where the CO₂ concentration may fall to a low value at which the error in log_e (CO₂ concentration) is high. Such experiments do not in fact reduce the variance of the air change, but rather tend to increase it (cf. Theoretical Analysis, § 3.6). It is of interest that in only one case is the 'interaction' variance greater than the 'internal variance', and in no case is it significantly greater. That is to say, the difference between the two rooms is constant within the errors of measurement. This indicates that the two rooms respond similarly to similar external conditions in each series of experiments, and that the difference between the two rooms under any set of conditions is approximately constant.

The error of method as measured by the difference between duplicates is probably not as great as that given above, since the duplicate is strictly less like

Table 7. *Duplicate air change. Analysis of variance*

Experimental conditions	Mean difference rooms 1-3	s.d. difference	Source of variation	Sum of squares	Degrees of freedom	Mean square	Variance ratio	Significance
Flueless rooms, 8 experiments	-0.086	0.088	Between experiments	0.291894	7	0.041699	6.14	Sig. 1% level
			Between rooms	0.029756	1	0.029756	4.38	Sig. 5% level
			'Interaction'	0.027094	7	0.003871	<1	Not sig.
			Total	0.348744	15	—	—	—
			'Internal variance'	—	32	0.006794	s.d. = 0.082	—
Rooms with flues, 7 experiments	-0.070	0.280	Between experiments	2.346343	6	0.391057	22.33	Sig. 1% level
			Between rooms	0.017150	1	0.017150	<1	Not sig.
			'Interaction'	0.233800	6	0.038967	2.22	Not sig.
			Total	2.597293	13	—	—	—
			'Internal variance'	—	28	0.017514	s.d. = 0.132	—
Rooms with fires, 4 x 15 min. experiments	+0.081	0.140	Between experiments	0.572237	3	0.190746	5.73	Sig. 1% level
			Between rooms	0.013613	1	0.013613	<1	Not sig.
			'Interaction'	0.029938	3	0.009979	<1	Not sig.
			Total	0.615788	7	—	—	—
			'Internal variance'	—	16	0.033270	s.d. = 0.182	—
Rooms with fires, 3 x 7½ min. experiments	-0.753	0.025	Between experiments	1.552300	2	0.776150	6.64	Sig. 5% level
			Between rooms	0.851267	1	0.851267	7.28	Sig. 5% level
			'Interaction'	0.000633	2	0.000317	<1	Not sig.
			Total	2.404200	5	—	—	—
			'Internal variance'	—	12	0.116888	s.d. = 0.341	—
Rooms with open windows	-0.27	0.202	Between experiments	12.715838	3	4.238613	36.84	Sig. 1% level
			Between rooms	0.143113	1	0.143113	1.24	Not sig.
			'Interaction'	0.062737	3	0.020912	<1	Not sig.
			Total	12.921688	7	—	—	—
			'Internal variance'	—	16	0.115052	s.d. = 0.339	—

The 'interaction' represents that part of the variance which is not included in the mean differences between rooms or between experiments. The term 'interaction' is used (Fisher, 1947, p. 92) since the sum of squares arises from the mean differences between rooms peculiar to the individual experiments. If it may be assumed that the differences between rooms and between experiments are strictly independent of one another, the 'interaction' may then be regarded as a measure of the error of air-change determination. In the present case, however, this is unnecessary, since the 'internal variance' has been used as an estimate of error which does not involve any such assumptions of independence: hence the 'interaction' has been compared with the 'internal variance' to test whether these differences are in fact independent.

the sample than is normally the case in experimental work. It is for this reason that the 'internal variance', which is independent of differences between the rooms, has been used in calculating the variance ratios.

The 'internal variance' contains the variability due to non-uniformity of mixing, to non-uniformity of fall of CO₂ concentration, and to the error in determination of CO₂ concentration. If mixing and fall of concentration are strictly uniform, the 'internal variance' will be due solely to errors in CO₂ determination. In this case, if four samples are taken at equal time intervals δ , the variance of air change q is that of $(dx/dt)_1$ given in equation (47) of the Theoretical Analysis. The standard deviation of CO₂ estimation is 0.036% (§5B), so that if the time interval is 5 min. and the CO₂ concentration about

2% (as in the case of many of our experiments) the standard deviation of air change q is about 0.10 per hour. The standard deviation so calculated is inversely proportional to CO₂ concentration and to the time interval between samples. In view of the variety of concentration of CO₂ used, this value is in tolerably good agreement with the standard deviation derived from the 'internal variance' of rooms with natural ventilation.

D. Errors arising from the mixing process

(i) Routine mixing error in a closed room

Experiments were done to ascertain the level of uniformity of mixing when two fans suitably sited were present in rooms 1 and 3 with doors, windows and cupboards closed. With two observers and with

the usual technique, eight points in the room (Text-fig. 2) were sampled, with the usual precautions, as rapidly as possible at various times after CO₂ had been liberated and thoroughly mixed. In order to minimize the error due to the time taken by one observer to collect the four samples (about 10 sec.), samples in consecutive experiments were taken in reverse order. The results of five experiments (A-E) are given in Table 8.

(ii) *Mixing error in a room with a high rate of air extraction*

In order to determine the mixing error in the presence of a rapid exit of air through the large orifice of a fire flue, two experiments were carried out in room 2 (of about 1100 cu.ft. capacity) containing an open fire burning briskly (5-6 air changes per hour). Mixing was maintained by two fans, one on either

Table 8. Concentration % of carbon dioxide at eight different points in a room

Exp.	Position in room								Mean	Sum of squares about mean
	1	2	3	4	5	6	7	8		
A	5.00	4.87	4.94	4.86	4.86	4.86	4.86	4.82	4.8812	0.0245
B	3.43	3.39	3.49	3.46	3.38	3.48	3.31	3.43	3.4212	0.0249
C	3.28	3.33	3.32	3.33	3.34	3.25	3.24	3.29	3.2975	0.0103
D	1.37	1.41	1.37	1.43	1.37	1.36	1.32	1.32	1.3688	0.0076
E	0.51	0.55	0.55	0.55	0.60	0.51	0.51	0.50	0.5287	0.0087

Sum of squares about the mean for 5 experiments = 0.0762.
 Variance (35 degrees freedom) = $\frac{0.0762}{35}$; s.d. = 0.047.

Comparing the s.d. from the above data with that obtained from duplicate estimation of CO₂ from the same sampling tube (§ 5 B) the 'F' test gives $F = 1.71$, $n_1 = 35$, $n_2 = 27$, showing that the two s.d.'s are not significantly different. We can hence assume that from the above data there is no appreciable error in estimation of the gas which can be assigned to routine mixing. Two similar experiments were done in a large room of volume 5000 cu.ft. In the first experiment (A) three fans were used (two usual fans and one rotating desk fan 12 in. diameter; 750 r.p.m.), and eight 'simultaneous' samples were taken. In the second experiment (B) two of the usual fans were used and seven 'simultaneous' samples were taken. The results are summarized below:

Exp.	...	A	B
Sum of estimations of concentration of CO ₂		20.24 (8)	19.54 (7)
Mean % CO ₂		2.53	2.79
Sum of squares about the mean		0.0228	0.0357

The estimate of variance based on all these readings combined (13 degrees of freedom) = 0.0045; s.d. = 0.067 % CO₂. This value is considerably greater than the s.d. for duplicates from one tube (0.036 %, see § 5 B). The variance ratio of these two s.d.'s is 0.0045/0.0013 = 3.46, with $n_1 = 13$, $n_2 = 27$ (1 % level for F is 2.88 for these degrees of freedom). It would thus appear that in large rooms the level of mixing as used above is not sufficient to maintain the uniformity of CO₂ concentration within the limits of the accuracy of CO₂ determination. However, the errors are small enough to permit reasonably accurate estimation of air change.

side of the fireplace, with a third fan in one opposite corner of the room. The fans were sited so as to give the maximum mixing of room air prior to its passage into the fire.

In the first experiment (A) seven 'simultaneous' samples were taken, and in the second experiment (B) eight samples were obtained. The results are summarized below:

Exp.	...	A	B
Sum of estimations of concentration of CO ₂		19.11 (7)	13.31 (8)
Mean % CO ₂		2.73	1.66
Sum of squares about the mean		0.0430	0.0366

The sums of squares may be combined to give an estimate of variance based on 13 degrees of freedom:

Variance = $\frac{0.0430 + 0.0366}{13} = 0.0061$,

giving an s.d. = 0.078 % CO₂. This value is more than twice the s.d. for duplicates (s.d. = 0.036 %). Using the 'F' test we have $F = 4.6$ with $n_1 = 13$ and $n_2 = 27$, giving a probability level of about 0.1 %. It thus appears that when there is a high rate of extraction of air from a small room, the degree of mixing applied above is not sufficient to keep the uniformity of CO₂ concentration within the limits of accuracy of CO₂ determinations. However, the errors are small enough to permit reasonably accurate estimates of the air change to be made as long as suitable time intervals are chosen so that the change in concentration is larger than the errors, without the concentration falling too low, and as long as a number of samples are taken so that the large error of a single difference is avoided.

(iii) *Mixing errors with large air inlets—windows*

A series of experiments was carried out in the experimental rooms under different conditions to ascertain the nature and degree of mixing present when windows were open. In all cases the usual mixing was initially employed with windows closed. These were opened and the first samples taken 1 min. later.

In general, samples taken within 1 ft. of an open window gave extremely fluctuating and unreliable results. Even if these points are omitted, the s.d. of several 'simultaneous' samples taken within the room was extremely high. As might be expected under the circumstances in which massive air movement is present, the errors appear to be larger at high CO₂ concentrations. In an experiment with a window (3½ × 1½ ft.) open, 'simultaneous' samples were taken at four different points in the room at 5 min. intervals with two fans in the usual positions. The combined s.d. for the mixing was 0.498% CO₂. In this case the omission of the samples near the window made

0.202% CO₂; on the other hand, comparison of this with mixing errors in closed rooms gives

$$F = \frac{0.0407}{0.0013} = 31.3, n_1 = 3, n_2 = 27,$$

this being highly significant. It is clear that the s.d. is very much higher than that of duplicates from the same sampling tube, but the level of accuracy is probably sufficient to get a rough estimate of the air change. A high level of accuracy under these conditions is of academic interest only.

A further experiment was carried out in which two hopper windows (10 × 18 in.) only were open, each having a fan deflecting air towards the ceiling, with the two usual fans present. The results gave a variance of CO₂ concentration of 0.0181 (s.d. = 0.134), hence $F = 0.0181/0.0013 = 13.9$, $n_1 = 11$, $n_2 = 27$, and this is highly significant. Here again the accuracy appears sufficient in order to obtain the estimate of air change required under these conditions.

A final experiment was carried out in a room of about 5000 cu.ft. with two open windows (3½ × 1½ ft.).

Table 9. *Concentration % of carbon dioxide in a room with open windows*

Position in room	Time of sampling (min.)				Air change per hour
	0	2½	5	7½	
(a) 1 ft. from window, table height	—	1.8	—	—	—
(b) 8 ft. from window, table height	2.05	1.71	1.07	0.93	6.83
(c) 12 ft. from window, table height	2.43	1.68	1.23	0.96	7.46
(d) 2 in. from floor	—	1.33	—	—	—
(e) 2 in. from ceiling	—	0.72	—	—	—

no appreciable difference to the s.d. The air change at the four points, derived from the data, gave a range of 3.7–5.7 air changes per hour. In a second experiment with the same mixing technique 'simultaneous' samples were taken at each of three points in the room at 5 min. intervals on four occasions. In this particular experiment four 'simultaneous' duplicate samples were also taken at the three points. The s.d. of the duplicates was found to be 0.017% CO₂ compared to 0.600% derived from the four series of three samples, and if the window samples were omitted the s.d. becomes 0.046% CO₂. It appears likely that the mixing errors under these conditions are due to considerable volumes of air maintaining their individuality unmixed, rather than to small local variations. Subsequently, measurements were made in which three fans were placed so as to mix air passing to or from the window, together with the two fans in the usual position. The samples taken near the window were still extremely variable, but four samples taken at each of two points in the body of the room showed better agreement. The mean difference between the CO₂ concentrations at these two points is not significant, with an s.d. of

In this experiment seven fans were used (six routine fans and one swivel fan 12 in. diameter; 750 r.p.m.), four of the routine fans being arranged in the hope of breaking up the air movement to and from the window. Four samples were taken at each of two points in the body of the room at 2½ min. intervals. In addition, three samples were taken on one occasion at other points in the room. The results are given in Table 9.

The samples taken in the body of the room (b and c) showed reasonably good agreement (variance = 0.033; s.d. = 0.183, 3 D.F.). Although the s.d. is significantly greater than duplicates from the same tube, there is probably sufficient basis to obtain an estimate of air change. Samples a, d, e indicate that such an estimate would be rather open to question, since the concentration at ceiling level is considerably less than half that in the body of the room. In all the above experiments external wind speeds were not greater than 9 m.p.h. It is obvious that depending upon the effect of level and variation of wind speed and direction, upon room volume, and the number and size of open windows, mixing errors may be much higher than those given above. It is

doubtful whether intensifying the mixing agency would always reduce such errors without affecting the conditions to be measured.

(iv) *Mixing error with people as the source of mixing*

Under normal conditions vigorous mixing of room air is absent, and for many experimental purposes it is desirable to eliminate it. An experiment was carried out in a closed room (room 1) containing four persons, in which the air and CO₂ were mixed initially in the usual way. Fans were then turned off. The four people then moved slowly round the room, and with the usual precautions eight 'simultaneous' samples were taken at eight different points in the room at 5 and 10 min. after mixing had been stopped. Although the CO₂ concentration was 5.4% in the first sample there was no tendency for 'stratification' (cf. §5D(v)), and mixing remained reasonably good. The variance of the series (16 readings; 2 means, 14 degrees of freedom) was 0.0238 and the s.d. 0.154% CO₂. This value is markedly higher than that of duplicates from one sampling tube. The 'F' test gives $F = 18.3$, $n_1 = 14$, $n_2 = 27$, showing the difference between the variances to be highly significant. It thus appears that mixing produced by the occupants of a living room is not as thorough as that obtained by fans.

Nevertheless, it would appear that air change can be estimated under these conditions and gives fairly reliable results if the experiments are sufficiently long for the changes of CO₂ concentration to be considerable and provided that the rate of air change is not too high, and there are no large air inlets.

(v) *Apparent 'stratification' of carbon dioxide in the absence of mixing*

The calculation of air change involves the assumption that the concentration of tracer substance is uniform throughout the enclosure, and that this concentration falls uniformly and exponentially. It is therefore of importance to determine whether these conditions do in fact prevail in the absence of mixing agent. In order to reduce the mixing effect of occupants of a room, samples were taken by the external method. The accuracy of external sampling of a point within an enclosure was found by simultaneous air sampling from within and outside a cubicle by the apparatus shown in Pl. 1, fig. 1, at CO₂ concentrations of 0.3–5.1%. The results were as follows:

Mean difference of CO₂ concentration = 0.002%;
s.d. difference = 0.059.

With 9 degrees of freedom $t = 0.111$; Not sig.

The error of external sampling is no greater than that of duplicates from the same sampling tube (cf. §5B).

A number of experiments were carried out in cupboards, a cubicle and in room 1, under conditions where temperature gradients were minimal, and where air movement was cut down by sealing off all large openings and obvious cracks. CO₂ was passed into the enclosures over a fan and mixed by one or more fans so as to produce a level of uniformity in air mixture as found in §5D(i). Mixing was continued for 2 min. after all the gas had been passed in and then stopped. 'Simultaneous' air samples were obtained by the external sampling method from three points in the middle of the enclosures at three vertical levels (2 in. from top, mid-level, and 2 in. from the floor). The first sample (time 0) was obtained within as short a period as possible after shutting off the fans—about $\frac{1}{2}$ –2 min. All manipulations were done from outside the enclosures. For each interval between sampling, the 'apparent air change' was calculated on the assumption that the usual method of estimation was valid. The results are given in Table 10. The effect of remixing is seen in Exp. 2 at 65 min. cursory examination of the table suggests an apparent separation of CO₂, to which the term 'apparent stratification' has been applied. It is clear that uniform mixing does not maintain itself in the absence of fans, and that there appears to be ventilation by downward displacement, modified slightly by diffusion, and the incoming air presumably rises to the top of the heavier air and CO₂ mixture. The surface of separation between the two tends to be effaced by diffusion as time proceeds, but 'stratification' seems to be maintained to some extent even up to an interval of 5 hr. (Exp. 4). In terms of ventilation the 'apparent air change' is low at all vertical levels except that at the 'surface of separation'. This level falls with time, at first rapidly and then slows off as the 'static head' becomes less. Such a process combined with diffusion decreases the peak rate of 'apparent air change' as the level of the 'surface of separation' falls in height. These phenomena are in accordance with the interpretation of equation (60) of the Theoretical Analysis.

Although in many cases the calculated air change is very low, the interval between sampling is long, the concentration of CO₂ high, and the difference in concentration definitely significant (cf. §5B). One may therefore assume that the differences in 'apparent air change' shown in Table 10 are real.

In Exp. 1 the 'apparent air change' at the top of the cupboard falls rapidly from an initially high level, while at the other levels the rate is reasonably constant at a comparatively low air change. The mid-level lies nearer the surface of separation, and shows higher figures than found at the bottom, where diffusion plays little part. The first hour of Exp. 2 shows approximately the same effect as given above. The figures from the second hour of this experiment

Table 10. 'Apparent stratification' of carbon dioxide in absence of mixing

Time	Concentration % CO ₂ at various levels			'Apparent air change' per hour				
	Top	Middle	Bottom	Top	Middle	Bottom		
Exp. 1. Room 1. Cupboard A (built in) (8 × 3½ × 1½ ft.)								
0 min.	10.15	11.48	11.67	} } } } }	5.05	0.88	0.57	
15 "	2.87	9.22	10.12		2.57	0.78	} } } }	
30 "	1.51	7.59	—		1.42	0.91		
45 "	1.06	6.05	7.17					
Air change with usual technique = 0.50 per hour (mean)								
Exp. 2. Room 1. Cupboard A								
0 min.	20.15	20.10	20.22	} } } }	5.33	1.15	0.68	
30 "	1.40	11.31	14.42		1.53	1.29	0.75	
60 "	0.65	5.94	10.00					
65 "	4.05	4.13	4.34					
67 "	3.45	3.66	3.85	} }	0.77	0.63	0.57	
127 "	1.61	1.95	2.16					
Air change with usual technique = 0.50 per hour (mean) Remixing from 60–65 min.								
Exp. 3. Cupboard B (office type: wood) (6 × 5 × 1½ ft.)								
0 min.	22.35	22.40	22.34	} } } }	0.65	0.14	0.11	
17 "	18.60	21.52	21.65		0.0	0.09	0.17	
32 "	18.59	21.00	20.75		0.70	0.08	0.07	
47 "	15.59	20.56	20.40		0.67	0.08	0.03	
62 "	13.17	20.17	20.24					
Air change with usual technique = 0.10 per hour (mean)								
Exp. 4. Cubicle (10 × 8 × 7 ft.)								
0 hr.	1.27	7.61	7.37	} } } } }	—	0.55	0.03	
1 "	0.98	4.37	7.13		—	0.86	0.22	
2 "	0.82	1.84	5.74		—	0.18	0.26	
3 "	1.54	1.53	4.43		—	0.07	0.36	
4 "	1.50	1.42	3.10		—	—0.09	0.37	
5 "	1.55	1.56	2.15					
Air change with usual technique = 0.35 per hour (mean)								
Exp. 5. Room 1								
0 min.	5.55	5.46	5.75	} } } }	1.37	0.84	0.19	
15 "	3.94	4.43	5.48		0.97	0.75	0.12	
31 "	3.04	3.63	5.31		} }	0.48	0.64	0.28
47 "	—	3.06	4.93			1.22	0.43	
65 "	2.31	2.12	4.33					
Air change with usual technique = 0.9 per hour (mean)								

indicate that even at the lower CO₂ concentrations involved, the effects of 'stratification' may not be negligible.

Exp. 3 shows no marked fall in CO₂ concentration except at the top, and the surface of separation has not reached either of the lower levels. In Exp. 4 the concentration at the top of the cubicle fluctuated erratically, apparently indicating that the diffusion of CO₂ from lower levels was roughly in equilibrium with ventilation processes. At the mid-level, the 'apparent air change' rose to a peak after 1 hr. and fell to a lower level after 2 hr. At the bottom of the cubicle the 'apparent air change' rises through-

out the experiment and appears to be near a peak at the end of 5 hr. Finally, in Exp. 5, the 'apparent air change' at the top level fell throughout the experiment, while that at the two lower levels rose during the same period. In the last two experiments there is the suggestion of a critical level of CO₂ concentration below which little further 'stratification' occurs between the top and the middle of the enclosure. At such concentrations the forces tending to cause stratification are presumably in equilibrium with those of convection. The critical concentration is lower in a cubicle where it is easier to maintain uniform conditions than in a room partly exposed to

external conditions of wind variation. In a cupboard where conditions are most uniform the concentration of CO₂ does not fall low enough to reach such an equilibrium. High rates of ventilation may be expected to reduce the critical concentration. The difference in concentration of CO₂ due to 'apparent stratification' encountered in the above series of experiments emphasizes the importance of adequate and continuous mixing during the whole of an experimental period.

(vi) *Effect of massive turbulence on air change*

Some degree of turbulence is required to maintain a uniformity of a gas mixture in a room. In order to ascertain whether a high level of turbulence within a room can itself influence ventilation, the following experiments were made.

The air changes in rooms 1 and 3 were estimated simultaneously with the rooms under the same internal conditions except for the power of the mixing agency. The results are given in Table 11 below. In Exps. 1 and 2 the fireplaces were sealed off in both rooms, and in Exps. 3 and 4 the fireplaces were open but no heating used. One room contained two fans in the usual positions, and the other room five similar fans together, with two centrifugal air pumps capable of delivering 6000 cu.ft./hr. (corresponding approximately to 6 air changes per hour). In the latter room air currents were so deflected that there was no direct air movement against any surface.

Table 11. *The effect of massive turbulence on air change*

Experiment	Air changes per hour	
	Massive turbulence	Usual turbulence
1	Room 3, 0.94	Room 1, 1.14
2	Room 1, 1.63	Room 3, 0.92
3	Room 1, 1.39	Room 3, 1.46
4	Room 3, 1.26	Room 1, 0.91

Mean difference massive - usual turbulence = + 0.20
 s.d. difference = 0.415
 $t = 0.964$
 Not sig.

Table 11 shows that only in Exp. 2 was the difference in air change more than the error of duplicate estimation. The mean difference in air change is not large in relation to the known error and is not statistically significant. It may hence be assumed that as long as there is no high velocity of air in the neighbourhood of openings, moderate increase in the mixing force is not likely to produce an appreciable effect upon air change.

E. *Air change and the production of carbon dioxide in occupied rooms*

(i) *The measurement of air change by means of the carbon dioxide produced by occupants. The Pettenkofer method*

When any gas not normally present in the atmosphere (or present only in small quantities) is produced at a steady rate in a room, the concentration gradually increases according to equation (19) of the Theoretical Analysis, until it reaches the steady state represented by equation (20), at which point the gas is extracted as rapidly as it is produced. From the time of Pettenkofer the CO₂ production of humans has been used in this way to measure ventilation rates. Standard text-books (e.g. Clay, 1947) still suggest that this is a suitable method for ventilation measurements. However, it is, in fact, open to considerable objections. The production of CO₂ by humans is a very variable quantity, and furthermore the concentration of CO₂ approaches its final equilibrium value exponentially, the exponent being the negative product of rate of air change and time. Provided the concentration of CO₂ in external air is negligible, at low rates of air change the time taken to reach this equilibrium within a reasonably close approximation is inordinately long. For example, at a ventilation rate of 1 air change per hour, the CO₂ concentration does not reach within 2% of its final concentration until 3.9 hr. after the occupancy has begun, and even if the error tolerated is 10% the time taken is 2.3 hr. At lower ventilation rates the time taken is correspondingly longer. Clearly it is difficult to maintain internal conditions constant over as long a period as this, and impossible to maintain external conditions. At higher rates of air change the time taken to reach equilibrium is shorter, but, on the other hand, the CO₂ level at equilibrium is lower and more difficult to measure accurately, while the correction due to CO₂ in incoming air is proportionately greater. If the rate of air change is 4 per hour, the CO₂ reaches within 10% of its final volume in about 35 min., and within 2% in less than an hour. Under these conditions, however, the final concentration is only about 0.1% higher than that of the incoming air, even under the crowded conditions represented by 150 cu.ft. of room volume per occupant. At lower occupation density the concentration reached is proportionately lower. Low concentrations of CO₂ (i.e. below 0.3%) are difficult to measure with any accuracy with the standard 10 c.c. Haldane apparatus (cf. §5B), and are subject to error due to the unknown composition of incoming air drawn from a variety of sources.

Alternatively, it might be supposed that the rate of air change could be estimated by the behaviour of the CO₂ concentration during its initial rise. During this period, however, it is only the curvature of the

graph relating CO₂ concentration with time which gives any information as to air change; the linear component depends on the rate of production and the room volume only. Smoothing the exponential curve is a real difficulty in the application of any such method. In terms of the expression for the rate of change of tracer gas concentration given in the Theoretical Analysis, equation (21), the factor $(p_{s0} - p'_s)$ is initially small and the factor e^{-qst} is initially unity, so that $(dp_s/dt)_{t=0} = w_s$. Only after some time will the other terms begin to assert themselves and tend to make the initial rise 'level off' and the factor q_s , which is to be measured begin to show some effect. To estimate the magnitude of this effect, CO₂ concentration must be measured with very much greater precision than that attained in our hands using the standard Haldane gas analysis apparatus. The portable Haldane CO₂ apparatus might be used if it is certain that the CO₂ concentration will not rise above its upper limit of 1.0% CO₂, as may be safely assumed in most circumstances. For routine work in ventilation studies with our technique, however, we considered this apparatus too laborious.

In two experiments (cf. Table 12) the rise of CO₂

Table 12. *The production of carbon dioxide in a room by eleven people*

Time (min.)	CO ₂ % in occupied room	
	Exp. 1, room 3 occupied	Exp. 2, room 1 occupied
0	0.16	0.20
10	0.26	0.26
20	0.36	0.36
30	0.46	0.41
40	0.51	0.57
50	0.56	0.62
Air change in control room	0.49/hr.	0.58/hr.

concentration with time was measured, once in room 1 and once in room 3. In each case the room in which the rise was measured was occupied by eleven people (ten seated and one observer); the other room was unoccupied and had air change estimated by the external method with sampling during the 50 min. period of the experiment, each room containing the usual two fans. The results are summarized in Table 12. In both cases there is initially a roughly linear rise in concentration, and then the rise begins to 'tail off'. Since the air-change measurement depends on this 'tailing off', it is clear from these figures that it is subject to large errors unless CO₂ concentration is determined to a very high degree of accuracy, and mixing errors eliminated. Probably the best estimate of air change is that given by the control room.

In sum, the Pettenkofer method may perhaps give a sufficiently reliable estimate in such rooms as

offices and dormitories where there is at least approximately statistical uniformity of occupancy and of internal conditions over long periods, and it is for such types of room that the method was originally designed. In dwelling houses these conditions do not prevail and it cannot be applied satisfactorily.

(ii) *Comparison of apparent rates of air change in occupied and unoccupied rooms*

It is clear that when CO₂ is used as a tracer gas in estimating rates of air change, the production of CO₂ by the occupants of the room will affect the measurement (cf. term v_s in the Theoretical Analysis equation (24)). In order to verify the magnitude of the effect, simultaneous experiments were performed in experimental rooms 1 and 3, one room containing a total of ten people (one observer) on light activity, and samples taken in the room. The other room was empty and air samples were taken by the external method. It has already been shown that there is no significant difference between internal and external sampling. To eliminate differences between rooms, between days, or between morning and afternoon experiments, the occupied room was 'balanced' with respect to these factors and within the limits so imposed, assignment was random. The experimental results are summarized in Table 13. For the uncorrected data in which the

Table 13. *Effect of ten individuals on the air change in a room*

Exp.	Time	Occupied room	Air change in	
			unoccupied room	Corrected air change in occupied room
1	a.m.	3	0.61	0.63
2	p.m.	1	0.83	0.45
3	a.m.	3	1.04	0.57
4	p.m.	1	0.90	0.50
5	a.m.	1	0.72	0.60
6	p.m.	3	0.75	0.49
7	a.m.	1	0.82	0.65
8	p.m.	3	0.87	0.83

air change is calculated as on p. 16, without correction for CO₂ production we have a mean difference in apparent air change between occupied and unoccupied rooms of 0.228 air changes per hour, and the variance of the difference is 0.0329 (s.d. = 0.179). Hence

$$t = \frac{0.230\sqrt{8}}{\sqrt{0.182}} = 3.590 \text{ (7 D.F.; Sig. 1 \%)}.$$

If, however, the correction is applied to allow for the production of CO₂ by the ten occupants at

a rate of 0.93 cu.ft. per head per hour (see Table, p. 12), the mean difference is greatly reduced to 0.016 air changes and the variance becomes 0.0438 (s.d. = 0.209). Hence

$$t = \frac{0.016}{0.209} \sqrt{8} < 1 \text{ (7 D.F.; Not sig.)}$$

The correction applied in Table 13 is v_c/p_c , where p_c is the mean CO₂ concentration during the experiment. This is a simplified form of the term $\frac{v_s - p'_s \sum v_s}{p_s - p'_s}$ in Theoretical Analysis equation (24), and it agrees surprisingly well with the observed difference. The s.d. of the difference is rather higher than would be expected from the s.d. of air-change measurement in closed rooms, but this may be accounted for in part by the fact that it includes the differences between rooms as well as the error of determination of air change (cf. §5C).

In sum, it appears that a correction term can be employed to give tolerably good correction for the error of air-change measurement due to the production of CO₂ (or any other tracer gas) by occupants. Unless the number of occupants is high in comparison with the size of the room, or the concentration of tracer gas is low, the correction is small. It is always positive in sign.

F. Errors arising from lack of uniformity during an experimental period

(i) Apparent fall in air change due to internal conditions in a room

For this analysis thirty-four experiments were used in which the concentration of CO₂ was measured at 0, 5, 10 and 15 min., and the rate of air change calculated on the assumption of logarithmic decrement of concentration during successive time intervals. The mean value for the calculated air change for the first 5 min., q_1 , of each experiment was rather higher than that for the last 5 min., q_3 , and the sign of the difference ($q_1 - q_3$) was positive in twenty-five cases out of thirty-four, and negative in nine only. Such a distribution of signs would occur only in about 1% of cases in random sampling with positive and negative signs equally distributed. The mean difference may be treated by the 't' test:

Sum of differences in thirty-four experiments

$$= \Sigma (q_1 - q_3) = +23.69$$

Mean difference = +0.70.

$$t = \frac{23.69}{34} \sqrt{\frac{34 \times 33}{73.6085}} = 2.72 \text{ (33 D.F.)}$$

This corresponds to a probability only slightly greater than 1% and appears to be significant. Similarly the ratio of the difference ($q_1 - q_3$) to the mean air change, \bar{q} , is significantly greater than zero;

giving $t = 2.68$ (33 D.F.) with probability slightly greater than 1%. A consistent effect of this kind is unlikely to be due to changes in external conditions of temperature or pressure which in this case can be regarded as random. However, we have the following factors to consider:

(1) The room and cupboards may form an 'interacting coupled system' (cf. Theoretical Analysis, §3.10) if the cupboards are imperfectly sealed. In this case the tracer gas will initially leak into the cupboards until the concentration in them reaches that of the room air. During this period of leakage the fall in tracer substance will be greater than the ventilation rate. Eventually, the concentration in the room will fall below that in the cupboards, and the gas leaking out will tend to maintain tracer gas concentration and reduce the calculated air change. Evidence is present in our data that such a leakage of tracer gas does in fact occur.

(2) The calculation of air change as used by us assumes that tracer gas concentration falls off to zero, whereas, in fact, the concentration in incoming air is not zero, and there is a correction corresponding to the term p'_s (cf. Theoretical Analysis, §3.2) and appearing as p'_s in equation (24). This introduces an error which reduces the apparent air change, and the error increases as the tracer gas concentration falls.

(3) If the room is occupied, production of the tracer gas (CO₂) by the occupants tends to maintain tracer gas concentration, and this becomes relatively more important as the concentration falls. Such an effect corresponds to the term K_s (cf. Theoretical Analysis, §3.2) which later becomes incorporated in the term v_s .

(4) There may be variation in rate of emission with concentration (cf. Theoretical Analysis, §3.3), due to absorption either by walls or furnishings at a rate increasing with concentration. If the rate of emission decreases with decreasing concentration, the effect on measured air change will be that observed. For example, we have data which suggest that in certain individuals at higher CO₂ concentrations (3-5%), small quantities of the gas may be temporarily stored in the body. However, such an effect is not of sufficient magnitude to be of any importance.

(5) The presence of CO₂ in the air raises its density, which may alter the ventilation rate, the increase being linearly dependent on the CO₂ concentration. This effect is discussed in Theoretical Analysis, §3.9. It will diminish as the CO₂ diminishes, so that the measured air change will change with time.

(6) Changes in room temperature necessitate a correction discussed in Theoretical Analysis, §3.7. If the temperature rises rapidly initially and then asymptotically approaches a final value, as would occur in rooms when first occupied or when heating

is turned on, there will be an apparent falling off of rate of air change. This effect, however, is very small and may be neglected.

Probably all the above factors contribute to the observed effect. The cupboard volume is about 5% of that of the rooms, and if the tracer gas leaks into the cupboards during the first 5 min. of an experiment, this would account for a fall in concentration of tracer gas of 5% of its initial value over a period of 5 min., or to 0.6 air change per hour approximately. In practice under experimental conditions, the CO₂ has been in the room some time before the first sample is taken, and only the final stage of the process would be observed. Over a period of 10 min. (as in the experiments in question) the difference ($q_1 - q_3$) (due to neglecting the concentration of CO₂ in incoming air) is about 0.005 air change per hour at an initial concentration of 1% and at an air change of 1 per hour.

Variation in rate of emission with concentration may be an important factor, but the associated constants are not known, so that it cannot be treated quantitatively. We calculate that the effect of the presence of one person in a room of 1000 cu.ft. producing CO₂ at the rate of 0.93 cu.ft. per hour (1 air change and 1% room CO₂) corresponds to a difference of only 0.018 air change in 10 min.

The effects most likely to account for the observed falling off of air change with time appear therefore to be those due to cupboards or those due to variation of ventilation rate with concentration (see Theoretical Analysis, §3.9). Since, however, there is no significant correlation between the difference ($q_1 - q_3$) and the tracer gas concentration ($r = -0.019$; Not sig.) or between the ratio $(q_1 - q_3)/\bar{q}$ and the concentration ($r = +0.18$; Not sig.) the evidence is inconclusive.

(ii) *Correlation with external wind conditions during consecutive five-minute periods*

The diurnal changes in ventilation rate of a room under fixed internal conditions are appreciable (cf. §5C) and appear to be correlated with changes in wind speed. It is possible that changes may occur over periods of time as short as those required for the experimental determination of air change, if there are marked changes of wind speed or direction.

During a number of experiments on natural ventilation in closed rooms, readings were taken on the anemometers outside the rooms at 1 min. intervals, beginning 2 min. before the ventilation measurement. Ventilation was measured by the usual technique using samples taken at 5 min. intervals. In this way it is possible to obtain the mean wind speed over time intervals simultaneous with, and 1 or 2 min. in advance of, the interval between successive samples of air. Correlation

coefficients (r) were calculated: (1) between change in ventilation rate during successive 5 min. intervals and change in wind speed over the same interval, and (2) between change in ventilation rate over successive 5 min. intervals and change in wind speed as read 2 min. before each sample. These correlation coefficients give a measure of short-term dependence of air-change fluctuations on wind fluctuations when the overall correlation of mean air change and wind is eliminated. In neither case was the correlation significant. Both correlations were negative, the larger giving $r = -0.26$ compared with the 5% probability level of $r = 0.29$ (43 D.F.). Any correlation which may be present must therefore be masked by the large error of estimation of air change from two CO₂ determinations at the beginning and end of a 5 min. interval. Change in wind direction has not been taken into account, and as pointed out by Carne (1946) the wind-flow patterns around a house are complex and difficult to evaluate.

Significant correlation over short periods would indicate that even over the length of an experiment the air change may vary with wind, and hence that even in comparatively short experiments the assumption of exponential decay was not justified. Since there is no significant correlation the ideal of exponential decay may be regarded as reasonable.

G. *Systematic error in air-change measurement*

(i) *Comparison of carbon dioxide and flowmeter methods*

A series of eleven experiments was done in a small cubicle (10 × 8 × 7 ft.) situated in a large laboratory under conditions where change in air temperature and movement were small. A calibrated flowmeter delivered outside air into the cubicle at various rates, and the air change was estimated by the CO₂ method. Preliminary experiments showed that with the use of fans and with suitable deflexion of air from the flowmeter, the level of mixing was of the order found in §5D(i). The 'natural' air change in the cubicle during the period over which the experiments were made varied from 0.31 to 0.42 per hour. The results of the eleven experiments at air changes from 0.5 to 8.0 per hour gave:

Mean difference flowmeter minus CO₂ methods
= 0.0045 air changes per hr.

s.d. difference = 0.435 air changes per hr.

$t < 1$; Not sig.

Another series of experiments in this room showed that at low rates of air change the difference between the two methods is negligible, but at rates above 5 per hour considerable differences begin to appear.

A further eight experiments were carried out in the experimental rooms. In room 1 various air changes (1–4.2) were produced by the flowmeter and

measured by the CO₂ method. Simultaneous 'natural' air change was measured in room 3, in the hope of throwing some light upon the difference between the methods. The results were as follows:

Mean difference flowmeter minus CO₂ methods
= 0.043 air changes per hr.

s.d. difference = 0.83 air changes per hr.

$t < 1$; Not sig.

In both series of experiments the mean differences were small, with a relatively high s.d. The means and s.d. were higher in the room than the cubicle, and the natural air change in room 3 threw little light on the difference in results between the methods. The latter is well outside the error of duplicate estimation of air change (cf. §5C).

Warner (1940), in a series of ten comparisons between a direct air-flow method and the katharometer with rates of air change between 1 and 22, found a mean difference of 0.06 air change, and a graph suggests a very close correlation between the methods. The comparisons were done in a galvanized iron tank of 56 l. (2 cu.ft.) capacity, so the results are strictly not applicable to ordinary rooms. However, considering the difficulty in maintaining adequate mixing at high air changes, the closeness of the agreement is surprising.

Marley (1935) carried out a series of seven comparisons between flowmeter and the katharometer methods. An analysis of his results shows that although the mean difference between the methods is zero, the s.d. of differences is 0.61 air change per hour, which is comparable with our results. It is of some interest that as conditions become progressively more realistic, a sealed metal box, a cubicle in a laboratory, and a room exposed to the weather, the difference between the methods increases.

It would appear that in spite of the theoretical relationship between the methods, there are appreciable differences and the problem is worthy of further study. Analysis of our own data and those of Marley, however, reveals the care that must be taken in drawing conclusions from small mean differences.

(ii) *Comparison of direct carbon dioxide (analysis) method with the katharometer method*

With the collaboration of the Building Research Station (Garston, Herts) a series of experiments was carried out in which simultaneous ventilation rates were determined in various rooms by the katharometer and direct CO₂ (analysis) methods.

The katharometers involved in the experiments were normally used for estimating an air-hydrogen mixture, and were modified to estimate the levels of CO₂ as used in this paper. It is hence possible that

the katharometer method as described below was not used at its optimum efficiency.

In the direct CO₂ (analysis) method, four gas samples were taken within the room at 2½ or 5 min. intervals, while in the case of the katharometer, readings were automatically recorded at 3 min. intervals and continued for the length of time laid down for the method (15–40 min.). Gas samples for both methods were taken as close as possible in the rooms. Otherwise the technique is that described in §4. The results of the various experiments are summarized in Table 14 below.

In the eleven experiments at rates of air change up to 2½ per hour the mean difference between the two methods is about 0.013 air change, the maximum difference is 0.22 air change and the standard deviation of the difference is 0.126 air change ($t < 1$; Not sig.). A scatter diagram plotting the difference between the two methods against the rate of air change by the direct CO₂ (analysis) method appears to indicate that, in this range, the katharometer gives rather higher readings than the direct CO₂ (analysis) method, and that the difference increases with the rate of air change. The results are not conclusive, however, and the differences are of too small an order to be of much more than academic interest.

At intermediate rates of air change, only three comparisons are available. As is to be expected, the differences are greater than at lower values of air change, but they are not large enough to be serious, and there does not appear to be any systematic effect. The mean difference between the two methods is about 0.19 air change (the katharometer giving the higher readings), the maximum difference is 0.43 air change and the s.d. of the difference is 0.27 air change ($t \sim 1.2$; Not sig.).

At very high rates of air change (10 air changes and over) the agreement is surprisingly good. In view of the difficulty in maintaining uniform mixing under these conditions and of the necessity of using fewer points in the determinations, some of them at the lower end of the scale, much greater differences might have been expected.

The conspicuous feature of the katharometer measurements is the very low variance of the air change, as calculated from the deviations from the 'least squares' straight line. In the case of the katharometer, however, the variance so calculated does not contain all the errors of a random nature; the zero line from which the katharometer deflexion is measured is found from preliminary measurements on uncontaminated air, the line through these recorded points being extrapolated; any error here will give rise to an error in air-change measurement. Such errors will probably be in part systematic, in part random, but neither part will contribute to the variance of the calculated air change except in so far

Table 14. Comparison of carbon dioxide (direct) method with the katharometer method

Exp.	Room	Katharometer method			Direct CO ₂ method			Difference katharometer—direct CO ₂ method
		Air change per hour	Variance of air change	s.D. of air change	Air change per hour	Variance of air change	s.D. of air change	
A	Room 4	1.92	0.007257	0.085	2.01	0.002894	0.054	-0.09
B	Room 5	1.83	0.003473	0.059	1.75	0.073008	0.270	+0.08
C	Room 6	1.27	0.000465	0.021	1.18	0.031464	0.177	+0.09
D	Room 7	1.44	0.001845	0.043	1.35	0.004709	0.069	+0.09
E	Room 8	4.37	0.006429	0.080	3.94	0.015638	0.125	+0.43
F	Room 9	1.77	0.011292	0.106	1.59	0.000216	0.015	+0.18
G	Room 10	3.05	0.000959	0.031	2.82	0.004867	0.070	+0.23
H	Roof space	11.22	0.042220	0.205	10.99	3.287	1.81	*
J	Room 11	3.59	0.006343	0.080	3.69	0.026755	0.164	-0.10
K	Corridor and hall	10.17	0.3667	0.606	13.51	0.6129	0.78	*
X1	Room 1	G 1.46	0.001423	0.038	1.38	0.002275	0.047	+0.08
		R 1.25	0.000788	0.028				-0.13
Y1	Room 1	G 1.07	0.004539	0.067	1.08	0.009821	0.099	-0.01
Y3	Room 3	R 0.72	0.001156	0.034	0.79	0.004838	0.070	-0.07
Z1	Room 1	R 1.05	0.003037	0.055	0.91	0.009547	0.098	+0.14
Z3	Room 3	G 0.64	0.000226	0.015	0.86	0.002160	0.046	-0.22

* Figures unreliable, s.D. large.

as it will introduce curvature to what ought to be a straight line; both will, however, affect the mean value of the slope of logarithmic decrement, and hence the air change. This may account for the larger differences between katharometer and direct CO₂ (analysis) methods of estimation, and also for the difference between simultaneous duplicate measurements of air change using two katharometers *G* and *R* as close together as possible (Exp. X1). Although this difference is of little practical importance, it is considerably greater than can be attributed to random error in CO₂ determination by the katharometer as indicated by the 'internal variance'.

Our figures show that we have been unable to reproduce the very close agreement found by Warner (1940) in his comparison of direct CO₂ (analysis) and katharometer methods, but our experiments were carried out under more realistic conditions and over a wider range of air change. Over the range of air change of our experiments, the differences between the two methods are probably no greater than may be expected in view of the assumptions involved, and are not likely to be of practical importance.

The katharometer has the advantage of ease in obtaining large numbers of observations and in increasing the length of an experiment. It is, moreover, well adapted to automatic readings, to remote control, and to multiple experiments. Neither method is sensitive at the low end of its range, and both methods are subject to considerable errors at very high rates of air change (greater than 10 per hour). In comparison with the katharometer the Haldane apparatus is clumsy, but it measures gas concentration directly and its errors can easily be traced.

6. DISCUSSION

The theoretical analysis has brought out the difficulty in formulating a precise conception of air change. On a rigid mathematical basis the term is elusive in the absence of perfect mixing, yet this is clearly absent under normal living conditions. If mixing is imposed, any measurement is of an artificial volume average, whereas many of the effects which are popularly associated with ventilation depend largely on local conditions. Even with perfect mixing, the simple definition (Theoretical Analysis, equation (7)) is meaningful only in an isolated system, yet under actual living conditions rooms form part of a coupled system involving the room and its cupboard spaces, a whole house or a block of flats and the external atmosphere at the time of observation. The importance of the coupling depends on the nature of the problem involved and the conditions in the different elements of the coupled system. For example, in a block of flats the overall ventilation of the whole block may be of principal interest in heating problems, but the ventilation of individual rooms is important from the point of view of health and comfort, and the extent to which coupling is relevant depends on the conditions in adjacent rooms. In household heating, the situation is intermediate, since heat passing from one room to another is not completely wasted, although it may have lost much of its value. In our experimental study, we have regarded the rooms in which air change has been measured as constituting completely closed systems, and have neglected all forms of coupling. In this case, difficulties due to coupling appear as a variation in the concentration of tracer gas in

incoming air; the latter is not necessarily external air, and it is obvious that corrections are uncertain. Errors of this nature, due to coupling between a room and cupboards within it, are discussed in the *Theoretical Analysis*, §3.10 and in §5F(i).

Difficulties due to coupling may be minimized by the use of hydrogen or helium, which do not enter into physiological exchanges to an appreciable extent and which are not formed in the process of combustion. Moreover, for the same reasons such tracer substances may be used in very low concentrations, so that density differences and the apparent tendency to 'stratify' are minimized (cf. §5D(v)). They have the drawback that the apparatus required for their estimation is more elaborate, and in the case of hydrogen or coal gas there are attendant dangers which CO₂ avoids. For routine use in dwelling-houses, under living conditions, the CO₂ method as used by us is probably the simpler.

The problem of mixing depends largely on the nature and rate of ventilation. Clearly perfect mixing is an ideal concept, and any mechanism which attempts to impose it makes the measurement to that extent artificial. In the past, a great deal of thought was given to the problem of the composition of the air in occupied rooms. Roscoe, in 1856, quoted figures obtained by Pettenkofer in lying-in wards bearing out the view, accepted since the time of Priestley, that vitiated air rose to the top of a room. Roscoe's own observations showed, however, that although in living rooms very small differences of CO₂ concentration in this direction occurred, in schoolrooms and barracks the higher concentrations of CO₂ were found near the floor. The differences quoted are small, however, and may consist largely of error of method. Rideal (1908) and Masterman *et. al.* (1935) found that in rooms where CO₂ was being given off by a flueless gas heater, the concentration of CO₂ was lower near the floor, but little variation was found above 1 ft. from floor level. It seems to us that the degree of uniformity of CO₂ in a room in which this gas is being produced depends upon the conditions in the room, and in particular upon the nature of the mixing agencies present (*Theoretical Analysis*, §3.8). It should be mentioned that although air movement in general helps mixing, unless it produces turbulence it will not necessarily mix efficiently. Most sources of CO₂ which are likely to be present in a room are also to some extent mixing agencies.

There has been little attempt in the recent literature to define the mixing agency, or to determine whether uniform mixing is in fact attained. Petri (1889) used papier maché fans, but stopped mixing during sampling, and tested efficiency of mixing by sampling at six points in a room. Marley (1935) suggested that after sufficient mixing, fans may be stopped during the experiment. With the CO₂ con-

centrations we have used 'apparent stratification' may sometimes occur under these conditions, but with hydrogen in the concentration customarily used this would be unimportant. Carne (1946), working with approximately 3% CO₂, used a fan for mixing, and sampled four points in the room. It is not clear whether mixing was stopped during the experiment, and since the four sampling points appear to have been at the same horizontal level, his technique would not have detected 'stratification' if this were present. Our own data clearly shows the importance of continuous mixing throughout the experiment, where this is possible without disturbing other factors relevant to the particular problem involved. Mixing becomes even more important at higher concentration of tracer gas. With substances of high molecular weight, such as are suggested by Lidwell & Lovelock (1946), diffusion is slow, and unless initial turbulence is thorough 'stratification' due to imperfect mixing may occur. The concentrations used are very low, however, so that once mixing is imposed initially density differences will be very small and further 'stratification' is unlikely to occur. In this respect the mixing precautions used with other gases should suffice. Nevertheless, such organic substances, especially those with polar molecules, are more likely to be absorbed by walls and clothing than are true gases used at temperatures well above their boiling-points. We have shown that in the presence of large openings in a room, mixing will be less thorough, particularly if air enters at high velocity. Although ventilation rates as high as 30 per hour in rooms with cross-ventilation are quoted (Bedford, Warner & Chrenko, 1943), it is difficult to imagine uniform mixing under these conditions. Such an air change represents an 'equivalent ventilation rate' at the point of sampling, but it may bear only a very rough relation to the air change of the room as a whole. High rates of air change have been considered of value in keeping down the bacterial content of room air, but the errors involved in measurement by tracer substances are large. Rates of air change as high as 10 per hour are likely to have much more meaning when they refer to conditions of well-distributed forced ventilation, as might be used in an operating theatre, than when they refer to free ventilation through large apertures, as often obtains in hospital wards. Although Warner (1940) found close agreement between katharometer and direct estimation methods up to 22 air changes per hour, the data were obtained from an experiment in a metal tank. The agreement may not be so good in a naturally ventilated room, and our comparison of flowmeter and direct CO₂ methods (cf. §5G(i)) suggests that considerable differences may occur.

A certain amount of mixing and turbulence is produced by any flueless heating appliance, and people in a room have a similar effect. Four people

moving in a room of about 1000 cu.ft. produced a surprising degree of uniformity of mixing, and the movement of occupants might be used for mixing tracer gas and air under conditions in which human comfort is also being measured and where the minimum disturbance of natural conditions is required. Although 'stratification' was found in an unoccupied room (cf. §5D(v)), on a different occasion in the same unoccupied room when the corridor heating was on and measurable convection currents were present, the tracer gas concentration was practically uniform. Uniform mixing, however, should not be assumed even in the presence of fans or other patent sources of turbulence, and an objective test should always be made. It has been suggested that the production of high air speeds by fans in a room might modify the ventilation rate, but we find that at the air speeds which we have used there is no evidence for errors of this type (cf. §5D(vi)). It has also been suggested that it is sufficient to mix thoroughly immediately before each sample is taken. This would be true if the outgoing air, even though not strictly uniform in composition, were a fair sample of the air within the room. Although the outgoing air may often be a sufficiently representative sample of the room air, this should be verified experimentally owing to the possibility of 'stratification' and ventilation by downward displacement suggested by our observations (cf. §5D(v)).

Although the statistical theory of errors is not new, its application in this type of experimental work is not yet widespread. The results shown in this paper suggest that the use of the standard Haldane gas analysis apparatus involves sensible error even at low gas concentrations. The effect of this error on air-change measurement is discussed in §5C. Previous writers on ventilation make little mention of the errors involved in the estimation of other tracer substances used. Such errors may be considerably less than those found by us, but it is preferable that they be known. One of the drawbacks of the use of CO₂ as a tracer gas is its universal presence. The production of CO₂ by the occupants of a room can be calculated and then converted into a correction term for air change. For one person in a room of about 1000 cu.ft., the correction may be neglected, but in small enclosures and at low air change and CO₂ concentration, it may be considerable. CO₂ production by heating appliances may be treated similarly. Corrections for CO₂ in the incoming air are generally small, since the concentration of tracer gas is high compared with that in external air, but since rooms are coupled systems part of the air may be drawn from other rooms in which CO₂ concentration may be unknown. Although many of the errors inherent in the CO₂ method are not found if hydrogen or coal gas is used, certain errors are nevertheless

present. Both these gases may be burnt in flueless gas heaters, and in a small kitchen the removal by such means is calculated to be equivalent to 0.20 air change per hour.

For convenience, we have used 5 min. as the sampling interval in experiments where the rate of air change is below 3 or 4 per hour. Calculation shows that the optimum sampling interval, using four samples taken at equal time intervals, is about 5½ min. at 4 air changes and is inversely proportional to the rate of air change (cf. Theoretical Analysis, §3.6). At low rates of air change strict adherence to the optimum would make experiments very long, and the rate of air change might well alter considerably during such an experiment. At low rates of air change the accuracy is probably as high as the assumptions involved warrant, even when 5 min. sampling is used.

It is of interest that a significant fall in the rate of air change may occur over as short a period as 15 min. This may be related to the internal conditions in the rooms, to coupling between cupboards and rooms, and to variation of the ventilation rate with CO₂ concentration. Although we have not found any significant correlation between air change over 5 min. intervals and external wind conditions, some connexion almost certainly exists. Clearly absolute uniformity during an experiment cannot be obtained, and it is not surprising that the 'internal variance' of air-change estimation is appreciable, and only in part due to the error of estimation of tracer gas concentration. The diurnal and day-to-day change in ventilation in closed rooms emphasizes the care necessary in interpreting the results of a series of experiments done in rooms where internal conditions are deliberately varied. It is possible that part of the differences observed are due to uncontrolled time variables.

It is often assumed that the measurement of air change carries with it a fair level of accuracy. However, our data show that in duplicate flueless rooms with a mean air change of 0.80 per hour, the limits of error as given by ± 2 s.d. of an estimate are ± 0.124 per hour or about $\pm 15\%$ of the mean. At rates up to 5 per hour the percentage error remains approximately the same, and such an order of error is likely to occur with the katharometer or other tracer methods. It has already been stressed that at rates over 10 per hour there is a rapid increase in mixing errors and the estimation becomes of dubious value.

Knowledge of error of method is necessary before significance can be attached to the result of a single experiment, and the technique of simultaneous duplicates in a designed experiment with a suitable statistical analysis may allow valid conclusions to be drawn from experiments in which complex interacting variables are present and where it is not possible to investigate the fine physical detail of the phenomena.

7. RÉSUMÉ

1. The various factors which influence the rate of change of any constituent of air in a room have been considered theoretically and their relevance to methods of measuring air change discussed. Mathematical expressions have been derived for the various physical relationships involved.

2. A technique is described where CO₂ is used as a tracer gas and liberated into a room from Douglas bags. Uniform concentration of the air mixture is maintained by electric fans. Gas samples can be obtained within or from outside the room, and the concentration of CO₂ measured in a standard Haldane gas analysis apparatus.

3. Using this apparatus the error in CO₂ measurement of concentrations up to 5% was estimated from duplicates and found to be

$$\text{s.d.} = 0.036\% \text{ CO}_2.$$

4. The error of method of air-change measurement was obtained from simultaneous duplicate estimations in similar rooms situated within a hut. Differences in the air temperature of these rooms and of wind speed outside when measured at the same time were either negligible or not significant. The error (± 2 s.d.) of the method is approximately $\pm 15\%$ of the mean with air change rates up to 5 per hour, but with higher rates difficulty in adequate mixing within a room greatly increases this error.

5. An investigation was made into the various errors modifying the uniformity of the air mixture when CO₂ is liberated into a room. In small closed flueless rooms (1000 cu.ft.) and using two small electric fans for air mixing, there is no significant mixing error, but in larger rooms (5000 cu.ft.) more fans are required to maintain a similar uniformity. In a small closed room containing a coal fire there is a small but significant mixing error. Small flueless rooms with open windows are associated with larger mixing errors, and with rates corresponding to 6 air changes per hour or more, experiments suggest that increase in the number and alteration in the position of the fans may not eliminate the very large mixing errors. If a uniform mixture is initially produced in a room, cessation of the mixing may lead to apparent 'stratification' of CO₂ and valid estimation of air change becomes impossible.

6. Experiments are described throwing light on the value of the Pettenkofer method of air-change measurement, using as tracer gas CO₂ produced by the occupants of a room. An error arises in occupied rooms in which air change is measured by the CO₂ method, due to the production of this gas by the occupants. Controlled experiments show that in a small flueless room with mean air change 0.8 per hour, the error due to the presence of ten occupants in light activity is: mean 0.23, s.d. 0.18 air change per hour.

7. Some errors arising from lack of uniformity during an experimental period are investigated.

8. The direct CO₂ method of air-change measurement as described in this paper is compared with the flowmeter method and that using a katharometer. In small closed flueless rooms and at air change rates up to 4.2 per hour, the flowmeter method gives results appreciably different from the CO₂ method (mean difference = 0.043; s.d. = 0.83 air change per hour). At air change rates up to 2.5 per hour and using CO₂ as a tracer substance, a good agreement is present between the Haldane and katharometer methods (mean difference = 0.013; s.d. = 0.126 air change per hour), but at higher rates of air change large differences appear.

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APPENDIX

Alphabetical list of symbols

a_s	Initial term in equation (10) for v_s , the rate of emission of the constituent s .
A_s	An abbreviation defined in equation (12); equal to a_s if p'_s is zero.
α	The constant term in equation (26); equal to $\log_e p_{s0}$.
b'_i	Coefficient of p'_i in equation (10).
b_{sr}	Coefficient of p_r in equation (10).
b_{ss}	Particular value of b_{sr} when $r=s$.
B_s	An abbreviation defined in equation (13).
\mathcal{B}	Wind strength on the Beaufort scale.
β	Coefficient of t in equation (26).
c	Suffix representing carbon dioxide.
C	Constant of integration equal to $P_0 p_{s0}/T_0$.
$d_1, d_2, d_3, d_5, d_6, d_9$	Functions of time-averages defined by equation (28).
D	An abbreviation for $(d_5 d_9 - d_6^2)$ defined by equation (28).
\mathcal{D}	Coefficient of eddy-diffusivity.
δ	Regular time-interval between observations.
Δp_{sn}	The difference ($p_{sn} - e^{sn}$) between an observed and a predicted concentration.
e	The base of Napierian logarithms, 2.7183...
E_n	An arbitrary coefficient, as used in equation (31).

$f()$	Symbol of implicit functionality, as in equation (43).	Q	Total volume rate at which room air is being expelled.
F	Ratio of two variances as used in Fisher's test.	Q'	Total volume rate at which atmospheric air is entering room.
g	The gravitational acceleration.	$Q_{\alpha\beta}$	Mutual ventilation coefficient or volume rate at which air passes from a space α to a space β , as in equation (64).
γ	Coefficient of t^2 in equation (26).	r	Human respiratory quotient; also used as a constituent suffix; and as a correlation coefficient.
h	Height of a man.	R	The general gas constant (equal to 1545 ft.lb./lb.mol./° F.)
H	Height of a room above a point at which air-composition is measured.	ρ_s	Absolute density or mass per unit volume of constituent s in a room.
k_s	Mass rate of generation per unit volume of constituent s at given point.	ρ'_s	Absolute density or mass per unit volume of constituent s in external atmosphere.
K_s	Total mass rate of generation of constituent s within a room.	s	Suffix representing a general constituent of the air.
m	Mass of a man (equation (50)); also used to denote a value of n that is not necessarily integral (equation (36)).	S	Surface area of a man's body.
M_s	Molecular weight of a constituent s of the air.	Σ	Indicating the operation of summation.
μ_s	Total mass of a constituent s present in a room.	t	Time; also used for the statistic of 'Students' test.
n	An ordinal number assuming the values 0, 1, 2, ..., N ; also used as a suffix denoting nitrogen; and as degrees of freedom.	t_n	Time at which the n th observation of a series is made.
N	Number of intervals between the $(N+1)$ observations in a series.	T	Absolute temperature of the air in a room.
o	Suffix denoting oxygen.	T'	Absolute temperature of external atmosphere.
p_s	Relative concentration or proportional partial pressure of constituent s in the air inside a room, equal to P_s/P .	T_α	Absolute temperature within a room α .
p'_s	Relative concentration or proportional partial pressure of constituent s in the external atmosphere, equal to P'_s/P' .	u	Vectorial velocity of air current at any point in a room.
p_{s0}, p'_{s0}	Initial values of proportional partial pressures of constituent s in room air and atmosphere respectively.	v_s	Equivalent number of room volumes (measured at room temperature and pressure) of the constituent s emitted per unit time, equal to RTK_{s0}/VPM_s .
$p_{s\infty}$	Ultimate steady-state value of proportional partial pressure of constituent s in room air.	$[v_s]$	Equivalent number of room volumes, reduced to N.T.P. conditions (32° F. and 760 mm. mercury), of the constituent s emitted per unit time.
$p_{s1}, p_{s2},$ etc.	Values of proportional partial pressures of constituent s in room air at times $t_1, t_2,$ etc.	V	Volume of a room.
P	Total pressure of the air in a room; also used for probability level.	w	Suffix denoting water vapour.
P_α	Total pressure of the air in a room α .	w_s	An abbreviation for a modified volume rate of emission of the constituent s defined in equation (18); equal to a_s if p'_s is zero (compare A_s).
P_0	Initial value of total pressure of the air in a room.	W	Rate of performance of external work by a man.
P'	Total pressure of the external atmosphere.	x	Abbreviation for $\log_e p_s$ (equation (25)).
P_s	Partial pressure of constituent s in room air.	x_n	Value of x at time t_n .
P'_s	Partial pressure of constituent s in external atmosphere.	y	Abbreviation for $\log_e(p_s - p'_s - w_s/g_s)$ (equation (23)).
P_s^n	A saturation partial pressure associated with an absorbent with respect to a constituent s .	z	A current variable.
q	The rate of air change expressed as the number of room volumes of room air (measured at room temperature and pressure) expelled in unit time, equal to Q/V .	\bar{z}	Mean value of z , equal to $\frac{1}{N+1} \sum_{n=0}^{n=N} z_n$.
q_s	An abbreviation for a mean modified rate of air change referred to a constituent s in accordance with equation (12).	z_n	Value of z at any time t_n .

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