5
Chiral perturbation theory

In Sec. 1.4 we discussed how to formulate an effective chiral Lagrangian for the self-interactions of low-momentum pseudo-Goldstone bosons, such as the pion. Chiral Lagrangians can also be used to describe the interactions of pions with hadrons containing a heavy quark. The use of chiral perturbation theory is valid for these interactions as long as the pion is soft, that is, has momentum $p \ll \Lambda_{\rm CSB}$. Chiral perturbation theory for heavy hadrons makes use of spontaneously broken $SU(3)_L \times SU(3)_R$ chiral symmetry on the light quarks, and spin-flavor symmetry on the heavy quarks. In this chapter, we study the implications of the combination of chiral and heavy quark symmetries for heavy hadron pion interactions.

5.1 Heavy mesons

In this section, we will obtain the chiral Lagrangian that describes the low-momentum interactions of the π, K, and η with the ground state $s_\ell = \frac{1}{2}$ spin symmetry doublet of heavy mesons, P_a and P_a^*. Some applications of the chiral Lagrangian are described later in this chapter. The chiral Lagrangian for other heavy hadron multiplets, e.g., heavy baryons, can be obtained similarly and is left to the problems at the end of the chapter.

As was noted in Chapter 2, we can combine the P_a and P_a^* fields into a 4×4 matrix,

$$H_a = \frac{(1 + \not{v})}{2}\left[P_a^{*\mu}\gamma_\mu + i P_a \gamma_5\right], \tag{5.1}$$

that transforms under the unbroken $SU(3)_V$ subgroup of chiral symmetry as an antitriplet

$$H_a \to H_b V_{ba}^\dagger, \tag{5.2}$$

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and transforms as a doublet

$$H_a \rightarrow D_Q(R) H_a, \tag{5.3}$$

under heavy quark spin symmetry. In Chapter 2, the fields P, P^* and the matrix H were also labeled by the flavor and velocity of the heavy quark. In this chapter, we are mainly concerned with the light quark dynamics, and so these labels are suppressed whenever possible.

The Lagrangian for the strong interactions of the P and P^* with low-momentum pseudo-Goldstone bosons should be the most general one consistent with the chiral and heavy quark symmetries defined in Eqs. (5.2) and (5.3), and it should contain at leading order the least number of derivatives and insertions of the light quark mass matrix. Fields such as P and P^*, which are not Goldstone bosons, are generically referred to as matter fields. Matter fields have a well-defined transformation rule under the unbroken vector $SU(3)_V$ symmetry, but they do not necessarily form representations of the spontaneously broken $SU(3)_L \times SU(3)_R$ chiral symmetry. To construct the chiral Lagrangian, it is useful to define an H field that transforms under the full $SU(3)_L \times SU(3)_R$ chiral symmetry group in such a way that the transformation reduces to Eq. (5.2) under the unbroken vector subgroup. The transformation of H under $SU(3)_L \times SU(3)_R$ is not uniquely defined, but one can show that all such Lagrangians are related to each other by field redefinitions and so make the same predictions for any physical observable.

For example, one can pick a field $\hat{H}_a$ that transforms as

$$\hat{H}_a \rightarrow \hat{H}_b L^\dagger_{ba}, \tag{5.4}$$

under chiral $SU(3)_L \times SU(3)_R$. This transformation property is a little unusual in that it singles out a special role for the $SU(3)_L$ transformations. The parity transform of $\hat{H}$ would then have to transform as in Eq. (5.4) but with L replaced by R. This forces upon us the following choice of parity transformation law:

$$P \hat{H}_a(\mathbf{x}, t) P^{-1} = \gamma^0 \hat{H}_b(-\mathbf{x}, t) \gamma^0 \Sigma_{ba}(-\mathbf{x}, t), \tag{5.5}$$

where Σ is the matrix defined in Eq. (1.99).

Clearly, Eq. (5.4) is not symmetric under $L \leftrightarrow R$, which causes the parity transformation rule to involve the Σ field. It is convenient to have a more symmetrical transformation for H. The key is to introduce a field

$$\xi = \exp(iM/f) = \sqrt{\Sigma}. \tag{5.6}$$

Because of the square root in Eq. (5.6), ξ transforms in a very complicated way under chiral $SU(3)_L \times SU(3)_R$ transformations,

$$\xi \rightarrow L \xi U^\dagger = U \xi R^\dagger, \tag{5.7}$$

where U is a function of L, R, and the meson fields $M(x)$. Since it depends on the meson fields, the unitary matrix U is space–time dependent, even though one is making a global chiral transformation with constant values for L and R. Under a $SU(3)_V$ transformation $L = R = V$, ξ has the simple transformation rule

$$\xi \to V\xi V^\dagger, \tag{5.8}$$

and

$$U = V. \tag{5.9}$$

The field

$$H_a = \hat{H}_b \xi_{ba} \tag{5.10}$$

transforms as

$$H_a \to H_b U^\dagger_{ba}, \tag{5.11}$$

under $SU(3)_L \times SU(3)_R$ transformations. Under parity

$$\begin{aligned} PH_aP^{-1} &= \gamma^0 \hat{H}_c \gamma^0 \Sigma_{cb} \xi^\dagger_{ba} \\ &= \gamma^0 H_a \gamma^0, \end{aligned} \tag{5.12}$$

which no longer involves Σ. For a generic matter field, it is convenient to use a field with a transformation law such as Eq. (5.11) that involves U but not L and R, and that reduces to the correct transformation rule under $SU(3)_V$. For example, if X is a matter field that transforms as an adjoint $X \to VXV^\dagger$ under $SU(3)_V$, one would pick the chiral transformation law $X \to UXU^\dagger$.

H and $\hat{H}$ lead to the same predictions for physical observables, since they are related by the field redefinition in Eq. (5.10),

$$H = \hat{H} + \frac{i}{f}\hat{H}M + \cdots, \tag{5.13}$$

which changes off-shell Green's functions but not S-matrix elements. In this chapter we use the H field transforming under $SU(3)_L \times SU(3)_R$ and parity as in Eqs. (5.11) and (5.12). Unless explicitly stated, traces are over the bispinor Lorentz indices and repeated $SU(3)$ indices (denoted by lower-case roman letters) are summed over.

Chiral Lagrangians for matter fields such as H are typically written with ξ rather than Σ for the Goldstone bosons. ξ has a transformation law that involves U, L, and R, whereas the matter field transformation law only involves U. In the construction of invariant Lagrangian terms, it is useful to form combinations of ξ whose transformation laws only involve U. Two such combinations with one derivative are

$$\begin{aligned} \mathbb{V}_\mu &= \frac{i}{2}(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger), \\ \mathbb{A}_\mu &= \frac{i}{2}(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger), \end{aligned} \tag{5.14}$$

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which transform under chiral $SU(3)_L \times SU(3)_R$ transformations as

$$\mathbb{V}_\mu \to U\mathbb{V}_\mu U^\dagger + iU\partial_\mu U^\dagger, \qquad \mathbb{A}_\mu \to U\mathbb{A}_\mu U^\dagger, \tag{5.15}$$

using the transformation rule in Eq. (5.7) for ξ. Thus $\mathbb{A}_\mu$ transforms like the adjoint representation under U and has the quantum numbers of an axial vector field, and $\mathbb{V}_\mu$ transforms like a U-gauge field and has the quantum numbers of a vector field. The $\mathbb{V}_\mu$ field can be used to define a chiral covariant derivative, $D_\mu = \partial_\mu - i\mathbb{V}_\mu$, that can be applied to fields transforming under U. Acting on a field F_a transforming like a **3** the covariant derivative is

$$(DF)_a = \partial F_a - i\mathbb{V}_{ab}F_b, \tag{5.16}$$

and acting on a field G_a transforming like a $\bar{\mathbf{3}}$ the covariant derivative is

$$(DG)_a = \partial G_a + iG_b\mathbb{V}_{ba}. \tag{5.17}$$

The H-field chiral Lagrangian is given by terms that are invariant under chiral $SU(3)_L \times SU(3)_R$ and heavy quark symmetry. The only term with zero derivatives is the H-field mass term $M_H \mathrm{Tr}\, \bar{H}_a H_a$. Scaling the heavy meson fields by $e^{-iM_H v\cdot x}$ removes this mass term. Once this is done, derivatives on the heavy meson fields produce factors of the small residual momentum, and the usual power counting of chiral perturbation theory applies. Scaling the H field to remove the mass term is equivalent to measuring energies in the effective theory relative to the H-field mass M_H, rather than m_Q.

The only allowed terms with one derivative are

$$\mathcal{L} = -i\mathrm{Tr}\, \bar{H}_a v_\mu \left(\partial^\mu \delta_{ab} + i\mathbb{V}^\mu_{ba}\right) H_b + g_\pi \mathrm{Tr}\, \bar{H}_a H_b \gamma_\nu \gamma_5 \mathbb{A}^\nu_{ba}. \tag{5.18}$$

Heavy quark spin symmetry implies that no gamma matrices can occur on the "heavy quark side" of $\bar{H}$ and H in the Lagrangian, i.e., between the two fields in the trace. Any combination of gamma matrices can occur on the light quark side, i.e., to the right of H in the trace. The H fields in Eq. (5.18) are at the same velocity, since low-momentum Goldstone boson exchange does not change the velocity of the heavy quark. It is easy to show that the only gamma matrices that give a nonvanishing contribution to $\mathrm{Tr}\, \bar{H}H\Gamma$ are $\Gamma = 1$ and $\Gamma = \gamma_\mu\gamma_5$. The $\Gamma = 1$ term was the H-field mass term discussed earlier. The $\Gamma = \gamma_\mu\gamma_5$ is the axial coupling to Goldstone bosons. Heavy quark symmetry symmetry implies that at leading order in $1/m_Q$, the coupling constant g_π is independent of the heavy quark mass, i.e., it has the same value for the D and $\bar{B}$ meson systems. The kinetic terms in Lagrange density Eq. (5.18) imply the propagators

$$\frac{i\delta_{ab}}{2(v\cdot k + i\varepsilon)}, \qquad \frac{-i\delta_{ab}(g_{\mu\nu} - v_\mu v_\nu)}{2(v\cdot k + i\varepsilon)}, \tag{5.19}$$

for the P_a and P^*_a mesons, respectively.

The terms in Lagrange density in Eq. (5.18) that arise from $\mathbb{V}_\nu$ contain an even number of pseudo-Goldstone boson fields, whereas the terms that arise from $\mathbb{A}_\nu$,

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and are proportional to g_π, contain an odd number of pseudo-Goldstone boson fields. Expanding $\mathbb{A}_\nu$, in terms of M, $\mathbb{A}_\nu = -\partial_\nu M/f + \cdots$, gives the P^*PM and P^*P^*M couplings

$$\mathcal{L}_{\text{int}} = \left(\frac{2ig_\pi}{f} P_a^{*\nu\dagger} P_b \partial_\nu M_{ba} + h.c\right) - \frac{2ig_\pi}{f} P_a^{*\alpha\dagger} P_b^{*\beta} \partial^\nu M_{ba} \varepsilon_{\alpha\lambda\beta\nu} v^\lambda. \tag{5.20}$$

The P^*PM and P^*P^*M coupling constants are equal at leading order in $1/m_Q$ as a consequence of heavy quark symmetry; the *PPM* coupling vanishes by parity. The coupling g_π determines the $D^* \to D\pi$ decay width at tree level

$$\Gamma(D^{*+} \to D^0\pi^+) = \frac{g_\pi^2 |\mathbf{p}_\pi|^3}{6\pi f^2}. \tag{5.21}$$

The width for a neutral pion in the final state is one-half of this, by isospin symmetry. The $B^* - B$ mass splitting is less than the pion mass so the analogous $B^* \to B\pi$ decay does not occur.

It is possible to systematically include effects that explicitly break chiral symmetry and heavy quark symmetry as corrections to the chiral Lagrangian. At the order of Λ_{QCD}/m_Q, heavy quark spin symmetry violation occurs only by means of the magnetic moment operator $\bar{Q}_v g\sigma^{\mu\nu} G_{\mu\nu}^A T^A Q_v$, which transforms as a singlet under $SU(3)_L \times SU(3)_R$ chiral symmetry, and as a vector under heavy quark spin symmetry. At leading order in the derivative expansion, its effects are taken into account by adding

$$\delta\mathcal{L}^{(1)} = \frac{\lambda_2}{m_Q} \text{Tr}\, \bar{H}_a \sigma^{\mu\nu} H_a \sigma_{\mu\nu} \tag{5.22}$$

to the Lagrange density in Eq. (5.18). The only effect of $\delta\mathcal{L}^{(1)}$ is to shift the masses of the P and P^* mesons, giving rise to the mass difference

$$\Delta^{(Q)} = m_{P^*} - m_P = -8\frac{\lambda_2}{m_Q}. \tag{5.23}$$

Including this effect, the P and P^* propagators are

$$\frac{i\delta_{ab}}{2\big(v\cdot k + 3\Delta^{(Q)}/4 + i\varepsilon\big)}, \quad \frac{-i\delta_{ab}(g_{\mu\nu} - v_\mu v_\nu)}{2\big(v\cdot k - \Delta^{(Q)}/4 + i\varepsilon\big)}, \tag{5.24}$$

respectively. In the rest frame $v = v_r$, an on-shell P has residual energy $-3\Delta^{(Q)}/4$ and an on-shell P^* has residual energy $\Delta^{(Q)}/4$. It is convenient when dealing with situations in which there is a real P meson and the P^* only appears as a virtual particle to rescale the heavy meson fields by an additional amount, $H \to e^{3i\Delta^{(Q)} v\cdot x/4} H$, so that the P and P^* propagators become

$$\frac{i\delta_{ab}}{2(v\cdot k + i\varepsilon)} \quad \text{and} \quad \frac{-i\delta_{ab}(g_{\mu\nu} - v_\mu v_\nu)}{2\big(v\cdot k - \Delta^{(Q)} + i\varepsilon\big)}, \tag{5.25}$$

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respectively. This rescaling is equivalent to measuring energies with respect to the pseudoscalar mass, rather than the average mass of the PP^* multiplet.

Chiral symmetry is explicitly broken by the quark mass matrix m_q, which transforms as $m_q \to L m_q R^\dagger$ under $SU(3)_L \times SU(3)_R$. Chiral symmetry breaking effects at lowest order are given by adding terms linear in m_q to the Lagrange density,

$$
\begin{aligned}
\delta\mathcal{L}^{(2)} = \sigma_1 \mathrm{Tr}\, \bar{H}_a H_b (\xi m_q^\dagger \xi + \xi^\dagger m_q \xi^\dagger)_{ab} \\
+ \sigma_1' \mathrm{Tr}\, \bar{H}_a H_a (\xi m_q^\dagger \xi + \xi^\dagger m_q \xi^\dagger)_{bb},
\end{aligned} \tag{5.26}
$$

where m_q is the light quark mass matrix. Expanding ξ in pion fields, $\xi = 1 + \cdots$, it is easy to see that the first term gives rise to mass differences between the heavy mesons due to $SU(3)_V$ breaking. The second term is an overall shift in the meson masses that is due to the light quark masses. It can be distinguished from the chirally symmetric term $\mathrm{Tr}\, \bar{H} H$ because it contains $\pi - H$ interaction terms. The σ_1' term is analogous to the σ term in pion–nucleon scattering. Both terms contain pion interactions of the pseudo-Goldstone bosons with the heavy mesons that do not vanish as the four momenta of the pseudo-Goldstone bosons go to zero, since they contain an explicit factor of chiral symmetry breaking.

The strange quark mass is not as small as the u and d quark masses, and predictions based just on chiral $SU(2)_L \times SU(2)_R$ typically work much better than those that use the full $SU(3)_L \times SU(3)_R$ symmetry group. The results of this section can be used for chiral $SU(2)_L \times SU(2)_R$, by restricting the flavor indices to 1–2, and using the upper 2×2 block of Eq. (1.100) for M, ignoring η. It is important to note that the parameters $g_\pi, \sigma_1, \sigma_1'$, and so on in the $SU(2)_L \times SU(2)_R$ chiral Lagrangian do not have the same values as those in the $SU(3)_L \times SU(3)_R$ chiral Lagrangian. The two-flavor Lagrangian can be obtained from the three-flavor Lagrangian by integrating out the K and η fields.

5.2 g_π in the nonrelativistic constituent quark model

The nonrelativistic constituent quark model is a phenomenological model for QCD in the nonperturbative regime. The quarks in a hadron are treated as nonrelativistic and interact by means of a potential $V(r)$ that is usually fixed to be linear at large distances and Coulombic at short distances. Gluonic degrees of freedom are neglected apart from their implicit role in giving rise to this potential and giving the light quarks their large constituent masses $m_u \simeq m_d \simeq 350$ MeV, $m_s \simeq 500$ MeV. This simple model predicts many properties of hadrons with surprising accuracy.

We use the quark model to compute the matrix element

$$
\langle D^+ | \bar{u} \gamma^3 \gamma_5 d | D^{*0} \rangle, \tag{5.27}
$$

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where the D^{*0} meson has $S_z = 0$ along the spin quantization $\hat{z}$ axis, and the heavy meson states are at rest. To calculate this transition matrix element, we need the operator $\bar{u}\gamma^3\gamma_5 d$ in terms of nonrelativistic constituent quark fields and the D^+ and D^{*0} state vectors. The decomposition of a quark field in terms of nonrelativistic constituent fields is

$$q = \begin{pmatrix} q_{\text{nr}}(\uparrow) \\ q_{\text{nr}}(\downarrow) \\ -\bar{q}_{\text{nr}}(\downarrow) \\ \bar{q}_{\text{nr}}(\uparrow) \end{pmatrix} + \cdots, \tag{5.28}$$

where the ellipses denote terms with derivatives. The field q_{nr} destroys a constituent quark and $\bar{q}_{\text{nr}}$ creates a constituent antiquark, with spins along the $\hat{z}$ axis as denoted by the arrow. (The lower two elements follow by acting with the charge conjugation operator on the upper two.) Using this decomposition, one finds

$$\begin{aligned} \bar{u}\gamma^3\gamma_5 d = \bar{u}^\dagger_{\text{nr}}(\downarrow)\bar{d}_{\text{nr}}(\downarrow) - \bar{u}^\dagger_{\text{nr}}(\uparrow)\bar{d}_{\text{nr}}(\uparrow) \\ + \text{terms involving quark fields.} \end{aligned} \tag{5.29}$$

In the matrix element Eq. (5.27), the overlap of spatial and color wave functions for the D and D^* states gives unity. The operator only acts nontrivially on the spin-flavor part of the state vector. In our conventions (see Chapter 2), $2i[S^3_Q, D] = -D^{*3}$, and this commutation relation fixes the relative phase of the D and D^* state vectors. Explicitly,

$$\begin{aligned} |D^{*0}\rangle &= |c\uparrow\rangle|\bar{u}\downarrow\rangle + |c\downarrow\rangle|\bar{u}\uparrow\rangle, \\ |D^{+}\rangle &= i(|c\uparrow\rangle|\bar{d}\downarrow\rangle - |c\downarrow\rangle|\bar{d}\uparrow\rangle). \end{aligned} \tag{5.30}$$

Equations (5.29) and (5.30) yield

$$\langle D^+|\bar{u}\gamma^3\gamma_5 d|D^{*0}\rangle = -2i, \tag{5.31}$$

where the heavy meson states at rest are normalized to two.

The matrix element in Eq. (5.27) can be related to the coupling g_π in the chiral Lagrangian by using the same method that was used in Sec. 1.7 to relate the parameter f to a matrix element of the axial current. Under an infinitesimal axial transformation

$$R = 1 + i\epsilon^B T^B, \qquad L = 1 - i\epsilon^B T^B, \tag{5.32}$$

the QCD Lagrange density changes by

$$\delta\mathcal{L}_{\text{QCD}} = -A^B_\mu \partial^\mu \epsilon^B, \tag{5.33}$$

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where A_μ^B is the axial current

$$A_\mu^B = \bar{q}\gamma_\mu\gamma_5 T^B q. \tag{5.34}$$

In Eq. (5.34), the $SU(3)$ generator T^B acts on flavor space, and color indices are suppressed. The transformation rule, $\Sigma \to L\Sigma R^\dagger$, implies that under the chiral transformation in Eq. (5.32) the pseudo-Goldstone boson fields transform as

$$\delta M = -f\epsilon^B T^B + \cdots, \tag{5.35}$$

where the ellipses denote terms containing M. Equations (5.4) and (5.10) imply that under an infinitesimal chiral transformation the change in the heavy meson fields vanishes up to terms containing the pseudo-Goldstone boson fields. Consequently the change in the effective chiral Lagrangian Eq. (5.18) under the infinitesimal axial transformation in Eq. (5.32) is

$$\delta\mathcal{L}_{\text{int}} = \left(2g_\pi i P_b^{*\nu} P_a^\dagger T_{ba}^B \partial_\nu \epsilon^B + \text{h.c.}\right) + 2ig_\pi P_a^{*\alpha\dagger} P_b^{*\beta} T_{ba}^B \epsilon_{\alpha\lambda\beta\nu} v^\lambda \partial^\nu \epsilon^B + \cdots. \tag{5.36}$$

Equating $\delta\mathcal{L}_{\text{int}}$ with $\delta\mathcal{L}_{\text{QCD}}$ implies that for matrix elements between heavy meson fields, the axial current can be written as

$$A_\mu^B = \left(-2ig_\pi P_{b\mu}^* P_a^\dagger T_{ba}^B + \text{h.c.}\right) - 2ig_\pi P_a^{*\alpha\dagger} P_b^{*\beta} T_{ba}^B \epsilon_{\alpha\lambda\beta\mu} v^\lambda + \cdots, \tag{5.37}$$

where the ellipses denote terms containing the pseudo-Goldstone boson fields. Using Eq. (5.37) leads to

$$\langle D^+ | \bar{u}\gamma^3\gamma_5 d | D^{*0}\rangle = -2ig_\pi, \tag{5.38}$$

and so the nonrelativistic constituent quark model predicts $g_\pi = 1$. Heavy quark flavor symmetry implies that it is the same g_π that determines both the $DD^*\pi$ and $BB^*\pi$ couplings. A similar result for the matrix element of the axial current between nucleons leads to the prediction $g_A = 5/3$ in the nonrelativistic constituent quark model, compared with the experimental value of 1.25. A recent lattice Monte Carlo simulation by the UKQCD Collaboration found $g_\pi = 0.42$ (G.M. de Divitiis et al., hep-lat/9807032).

5.3 $\bar{B} \to \pi e\bar{\nu}_e$ and $D \to \pi\bar{e}\nu_e$ decay

The decay rates for $\bar{B} \to \pi e\bar{\nu}_e$ and $D \to \pi\bar{e}\nu_e$ are determined by the transition matrix elements,

$$\langle\pi(p_\pi)|\bar{q}_a\gamma_\mu(1-\gamma_5)Q\big|P^{(Q)}(p_P)\big\rangle = f_+^{(Q)}(p_P + p_\pi)_\mu + f_-^{(Q)}(p_P - p_\pi)_\mu. \tag{5.39}$$

Here $f_-^{(Q)}$ can be neglected since its contribution is proportional to the lepton mass in the decay amplitude. The form factors are usually considered to be

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functions of $q^2 = (p_P - p_\pi)^2$. However, here it is convenient to view the form factors $f_+^{(Q)}$ and $f_-^{(Q)}$ as functions of $v \cdot p_\pi$, where $p_P = m_P v$. The right-hand side of Eq. (5.39) can be rewritten as

$$\left[f_+^{(Q)} + f_-^{(Q)}\right] m_P v_\mu + \left[f_+^{(Q)} - f_-^{(Q)}\right] p_{\pi\,\mu}. \tag{5.40}$$

In the region of phase space where $v \cdot p_\pi \ll m_Q$, momentum transfers to the light degrees of freedom are small compared with the heavy quark mass, and a transition to HQET is appropriate. Apart from logarithms of m_Q in matching the left-handed current onto the corresponding HQET operator, the left-hand side of Eq. (5.39) depends on m_Q only through the normalization of the $P^{(Q)}$ state, and so it is proportional to $\sqrt{m_Q}$. This gives the following scaling for large m_Q:

$$\begin{aligned} f_+^{(Q)} + f_-^{(Q)} &\sim \mathcal{O}(1/\sqrt{m_Q}), \\ f_+^{(Q)} - f_-^{(Q)} &\sim \mathcal{O}(\sqrt{m_Q}), \end{aligned} \tag{5.41}$$

and in the limit $m_Q \to \infty$, $f_+^{(Q)} = -f_-^{(Q)}$.

Neglecting perturbative corrections, we find the relation between B and D form factors is

$$\begin{aligned} f_+^{(b)} + f_-^{(b)} &= \sqrt{\frac{m_D}{m_B}}\left[f_+^{(c)} + f_-^{(c)}\right], \\ f_+^{(b)} - f_-^{(b)} &= \sqrt{\frac{m_B}{m_D}}\left[f_+^{(c)} - f_-^{(c)}\right], \end{aligned} \tag{5.42}$$

where in Eqs. (5.42) the form factors for $Q = b$ and $Q = c$ are evaluated at the same value of $v \cdot p_\pi$. Since the decay rate is almost independent of $f_-^{(Q)}$, it is more useful to have a relation just between $f_+^{(b)}$ and $f_+^{(c)}$. Using $f_+^{(Q)} = -f_-^{(Q)}$ in Eq. (5.42) yields such a relation:

$$f_+^{(b)} = \sqrt{\frac{m_B}{m_D}}\, f_+^{(c)}. \tag{5.43}$$

Equation (5.43) relates the decay rates for $\bar{B} \to \pi e \bar{\nu}_e$ and $D \to \pi \bar{e} \nu_e$ over the part of the phase space where $v \cdot p_\pi \ll m_Q$.

An implicit assumption about the smoothness of the form factors was made in deriving Eq. (5.43). We shall see that this assumption is not valid for very small $v \cdot p_\pi$. In this kinematic region chiral perturbation theory can be used to determine the amplitude.

The operator $\bar{q}_a \gamma^\nu (1 - \gamma_5) Q_v$ transforms as $(\bar{\mathbf{3}}_L, \mathbf{1}_R)$ under $SU(3)_L \times SU(3)_R$ chiral symmetry. This QCD operator is represented in the chiral Lagrangian by an operator constructed out of H and ξ with the same quantum numbers. At

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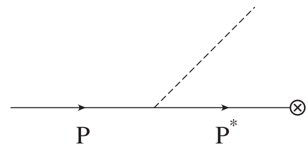

Fig. 5.1. Pole graph contribution to the heavy meson decay form factors. The axial current insertion is denoted by $\otimes$. The $PP^*\pi$ coupling is from the g_π term in the chiral Lagrangian.

zeroth order in the derivative expansion, it has the form

$$\bar{q}_a \gamma^\nu (1-\gamma_5) Q_v = \frac{a}{2} \mathrm{Tr}\, \gamma^\nu (1-\gamma_5) H_b\, \xi^\dagger_{ba}. \tag{5.44}$$

Heavy quark symmetry has been used to restrict the form of the right-hand side. Operators with derivatives and/or insertions of the light quark mass matrix m_q are higher order in chiral perturbation theory. Recall that $\xi = \exp(iM/f) = 1 + \cdots$, so the part of Eq. (5.44) independent of the pseudo-Goldstone boson fields annihilates P and P^*. This term was already encountered in Sec. 2.8 when we studied the meson decay constants f_D and f_B. At $\mu = m_Q$, $a = \sqrt{m_{P^{(Q)}}}\, f_{P^{(Q)}}$. The part of Eq. (5.44) that is linear in the pseudo-Goldstone boson fields contributes to the $P^{(Q)} \to \pi$ matrix element of Eq. (5.44). There is another contribution from the Feynman diagram in Fig. 5.1 that is also leading order in chiral perturbation theory. Here the $P^{*(Q)} P^{(Q)} \pi$ coupling has one factor of momentum p_π, but that is compensated by the $P^{*(Q)}$ propagator, which is of the order of $1/p_\pi$. The direct and pole contributions together give

$$\begin{aligned} f_+^{(Q)} + f_-^{(Q)} &= \left[\frac{f_{P^{(Q)}}}{f}\right]\left[1 - \frac{g_\pi v \cdot p_\pi}{v \cdot p_\pi + \Delta^{(Q)}}\right], \\ f_+^{(Q)} - f_-^{(Q)} &= \frac{g_\pi f_{P^{(Q)}} m_{P^{(Q)}}}{f\left[v \cdot p_\pi + \Delta^{(Q)}\right]}. \end{aligned} \tag{5.45}$$

Note that $f_+^{(Q)} - f_-^{(Q)}$ is enhanced by $m_{P^{(Q)}}/v \cdot p_\pi$ over $f_+^{(Q)} + f_-^{(Q)}$ and so $f_+^{(Q)} \simeq -f_-^{(Q)}$. Using this relation, we find the prediction of chiral perturbation theory for $f_+^{(Q)}$ becomes

$$f_+^{(Q)} = \frac{g_\pi f_{P^{(Q)}} m_{P^{(Q)}}}{2f\left[v \cdot p_\pi + \Delta^{(Q)}\right]}. \tag{5.46}$$

For $v \cdot p_\pi \gg \Delta^{(b,c)}$ the scaling relation between $f_+^{(b)}$ and $f_+^{(c)}$ in Eqs. (5.43) holds if $1/m_Q$ corrections in the relation between f_B and f_D are small. However, for pions almost at rest, Eq. (5.43) has large corrections because m_π is almost equal to $\Delta^{(c)}$. The derivation of Eq. (5.46) only relies on chiral $SU(2)_L \times SU(2)_R$ symmetry, and it is not necessary to assume that the strange quark mass is also small.

Using chiral $SU(3)_L \times SU(3)_R$, a formula similar to Eq. (5.46) holds for the decay $D \to K\bar{e}\nu_e$. Experimental data on the $D \to K\bar{e}\nu_e$ differential decay rate indicate that $f_+^{(D\to K)}(q^2)$ is consistent with the pole form

$$f_+^{(D\to K)}(q^2) = \frac{f_+^{(D\to K)}(0)}{1 - q^2/M^2}, \tag{5.47}$$

where $M = 2.1$ GeV. With this form for $f_+^{(D\to K)}(q^2)$, the measured decay rate implies that $|V_{cs} f_+^{(D\to K)}(0)| = 0.73 \pm 0.03$. Using $|V_{cs}| = 0.94$, we find this implies that at zero recoil, i.e., $q^2 = q^2_{\max} = (m_D - m_K)^2$, the form factor has the value $|f_+^{(D\to K)}(q^2_{\max})| = 1.31$. The zero-recoil analog of Eq. (5.46) for this situation is

$$\frac{g_\pi f_{D_s} m_{D_s}}{2f\left(m_K + m_{D_s^*} - m_D\right)} = f_+^{(D\to K)}\left(q^2_{\max}\right). \tag{5.48}$$

With the use of the experimental value for $f_+^{(D\to K)}(q^2_{\max})$, this implies that $g_\pi f_{D_s} = 129$ MeV. For the lattice value of f_{D_s} in Table 2.3, this gives $g_\pi = 0.6$.

5.4 Radiative D^* decay

The measured branching ratios for D^* decay are presented in Table 5.1. The decay $D^{*0} \to D^+\pi^-$ is forbidden, since $m_{\pi^-} > m_{D^{*0}} - m_{D^+}$. For D^{*0} decay, the electromagnetic and hadronic branching ratios are comparable. Naively, the electromagnetic decay should be suppressed by α compared with the strong one. However, in this case the strong decay is phase space suppressed since $m_{D^*} - m_D$ is very near m_π. For D^{*+} decays, the electromagnetic branching ratio is smaller

Table 5.1. *Measured branching ratios for radiative D^* decay*[a]

Decay mode	Branching ratio(%)
$D^{*0} \to D^0\pi^0$	61.9 ± 2.9
$D^{*0} \to D^0\gamma$	38.1 ± 2.9
$D^{*+} \to D^0\pi^+$	68.3 ± 1.4
$D^{*+} \to D^+\pi^0$	30.6 ± 2.5
$D^{*+} \to D^+\gamma$	1.7 ± 0.5
$D_s^{*+} \to D_s^+\pi^0$	5.8 ± 2.5
$D_s^{*+} \to D_s^+\gamma$	94.2 ± 2.5

[a] The branching ratio for $D^{*+} \to D^+\gamma$ is from a recent CLEO measurement (J. Bartlet et al., Phys. Rev. Lett. 80, 1998, 3919).

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than for the D^{*0} case because of a cancellation that we will discuss shortly. The decay $D_s^{*+} \to D_s^+ \pi^0$ is isospin violating and its rate is quite small.

The $D_a^* \to D_a \gamma$ matrix elements have the form [a is an $SU(3)$ index so $D_1 = D^0$, $D_2 = D^+$ and $D_3 = D_s^+$]

$$\mathcal{M}(D_a^* \to D_a \gamma) = e\mu_a \epsilon^{\mu\alpha\beta\lambda} \epsilon_\mu^*(\gamma) v_\alpha k_\beta \epsilon_\lambda(D^*), \tag{5.49}$$

where $\epsilon(\gamma)$ and $\epsilon(D^*)$ are the polarization vectors of the photon and D^*, v is the D^* four velocity (we work in its rest frame where $v = v_r$), and k is the photon's four momentum. The factor $e\mu_a/2$ is a transition magnetic moment. Equation (5.49) yields the decay rate

$$\Gamma(D_a^* \to D_a \gamma) = \frac{\alpha}{3} |\mu_a|^2 |\mathbf{k}|^3. \tag{5.50}$$

The $D_a^* \to D_a \gamma$ matrix elements get contributions from the photon coupling to the light quarks through the light quark part of the electromagnetic current, $\frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s$, and the photon coupling to the charm quark through its contribution to the electromagnetic current, $\frac{2}{3}\bar{c}\gamma_\mu c$. The part of μ_a that comes from the charm quark part of the electromagnetic current $\mu^{(h)}$ is fixed by heavy quark symmetry. The simplest way to derive it is to examine the $D^* \to D$ matrix element of $\bar{c}\gamma_\mu c$ with the recoil velocity of the D being given approximately by $v' \simeq (1, -\mathbf{k}/m_c)$. Linearizing in $\mathbf{k}$, and using the methods developed in Chapter 2, the heavy quark symmetry prediction for this matrix element is

$$\mu^{(h)} = \frac{2}{3m_c}, \tag{5.51}$$

which is the magnetic moment of a Dirac fermion. Another way to derive Eq. (5.51) is to include electromagnetic interactions in the HQET Lagrangian. Then $\mu^{(h)}$ comes from the order Λ_{QCD}/m_c magnetic moment interaction analgous to the chromagnetic term discussed in Chapter 4. The part of μ_a that comes from the photon coupling to the light quark part of the electromagnetic current is denoted by $\mu_a^{(\ell)}$. It is not fixed by heavy quark symmetry. However, the light quark part of the electromagnetic current transforms as an $\mathbf{8}$ under the unbroken $SU(3)_V$ group, while the D and D^* states are $\bar{\mathbf{3}}$'s. Since there is only one way to combine a $\mathbf{3}$ and a $\bar{\mathbf{3}}$ into an $\mathbf{8}$, the three transition magnetic moments $\mu_a^{(\ell)}$ are expressible in terms of a single reduced matrix element β,

$$\mu_a^{(\ell)} = Q_a \beta, \tag{5.52}$$

where $Q_1 = 2/3$, $Q_2 = -1/3$ and $Q_3 = -1/3$.

Equation (5.52) is a consequence of $SU(3)_V$ symmetry. Even the relation between $\mu_1^{(\ell)}$ and $\mu_2^{(\ell)}$ depends on $SU(3)_V$ symmetry. The contribution of u and d quarks to the electromagnetic current is a combination of $I = 0$ and $I = 1$ pieces, and so isospin symmetry alone does not imply any relation between $\mu_1^{(\ell)}$ and $\mu_2^{(\ell)}$. We expect that $SU(3)_V$ violations are very important for $\mu_a = \mu^{(h)} + \mu_a^{(\ell)}$. This

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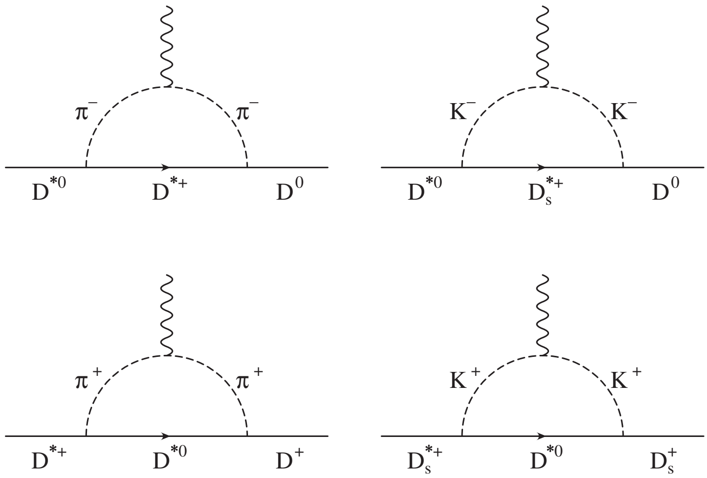

Fig. 5.2. The order $m_q^{1/2}$ corrections to the radiative D^* decay amplitude.

expectation is based on the nonrelativistic constituent quark model. In that model, the $\bar{u}$, $\bar{d}$, and $\bar{s}$ quarks in a D or D^* meson are also treated as heavy, and their contribution to $\mu_a^{(\ell)}$ can be determined in the same way that the charm quark contribution $\mu^{(h)}$ was. This yields

$$\mu_1^{(\ell)} = \frac{2}{3}\frac{1}{m_u}, \quad \mu_2^{(\ell)} = -\frac{1}{3}\frac{1}{m_d}, \quad \mu_3^{(\ell)} = -\frac{1}{3}\frac{1}{m_s}. \tag{5.53}$$

The large $SU(3)_V$ violations occur because for the usual values of the constituent quark masses $m_u \simeq m_d = 350$ MeV, $m_s = 500$ MeV and $m_c = 1.5$ GeV, $\mu_2^{(\ell)}$ and $\mu_3^{(\ell)}$ almost cancel against $\mu^{(h)}$. This cancellation is consistent with the suppression of the $D^{*+} \to D^+\gamma$ rate evident in Table 5.1. With the constituent quark masses given above, the nonrelativistic quark model predictions for the μ_a are $\mu_1 \simeq 2.3\ \text{GeV}^{-1}$, $\mu_2 = -0.51\ \text{GeV}^{-1}$, and $\mu_3 = -0.22\ \text{GeV}^{-1}$.

In chiral perturbation theory the leading $SU(3)_V$ violations are of the order of $m_q^{1/2}$ and come from the Feynman diagrams in Fig. 5.2. The diagrams are calculated with initial and final heavy mesons at the same four velocity v, but the final state D has a residual four momentum $-k$. These diagrams give contributions to μ_a of the order of $m_{(\pi,K)}/f^2$, and their nonanalytic dependence on m_q ensures that higher-order terms in the chiral Lagrangian do not give rise to such terms.

For the Feynman diagrams in Fig. 5.2 to be calculated, the chiral Lagrangian for strong interactions of the pseudo-Goldstone bosons in Eq. (1.102) must be gauged with respect to the electromagnetic subgroup of $SU(3)_V$ transformations. This is done by replacing a derivative of Σ with the covariant derivative

$$\partial_\mu \Sigma \to D_\mu \Sigma = \partial_\mu \Sigma + ie[Q, \Sigma]\mathcal{A}_\mu, \tag{5.54}$$

where

$$Q = \begin{bmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{bmatrix}, \tag{5.55}$$

and $\mathcal{A}$ is the photon field. The electromagnetic interactions arise on gauging a $U(1)$ subgroup of the unbroken $SU(3)_V$ symmetry. Since ξ transforms the same way as Σ under $SU(3)_V$, the covariant derivative of ξ is $D_\mu \xi = \partial_\mu \xi + ie[Q, \xi]\mathcal{A}_\mu$.

The strong and electromagnetic interactions are described at leading order in chiral perturbation theory by the Lagrangian

$$\mathcal{L}_{\text{eff}} = \frac{f^2}{8} \text{Tr}\, D^\mu \Sigma (D_\mu \Sigma)^\dagger + v \text{Tr}(m_q \Sigma + m_q \Sigma^\dagger), \tag{5.56}$$

where in this case the trace is over light quark flavor indices. It gives rise to the $MM\gamma$ interaction term

$$\mathcal{L}_{\text{int}} = ie\mathcal{A}_\mu \{[Q, M]_{ab} \partial^\mu M_{ba}\}. \tag{5.57}$$

Using the Feynman rules that follow from Eqs. (5.20) and (5.57), we find the last diagram in Fig. 5.2 gives the following contribution to the $D_s^{*+} \to D_s^+ \gamma$ decay amplitude:

$$\begin{aligned} \delta\mathcal{M} &= i \int \frac{d^n q}{(2\pi)^n} \left(\frac{2}{f} g_\pi \epsilon_{\alpha\lambda\beta\nu} v^\lambda q^\nu \right) \left(\frac{2g_\pi}{f} k^\eta \right) \\ &\quad \times \frac{g^{\alpha\eta}}{2v \cdot q} (e2q_\mu) \left(\frac{1}{q^2 - m_K^2} \right)^2 \epsilon^\beta (D_s^*) \epsilon^\mu (\gamma) \\ &= \frac{4ig^2 e}{f^2} \epsilon_{\alpha\lambda\beta\nu} v^\lambda k^\alpha \epsilon^\beta (D_s^*) \epsilon_\mu^* (\gamma) \int \frac{d^n q}{(2\pi)^n} \frac{q^\nu q^\mu}{\left(q^2 - m_K^2\right)^2 v \cdot q}. \end{aligned} \tag{5.58}$$

In Eq. (5.58) only the linear dependence on k has been kept. The second term in large parentheses is the $D_s^* DK$ coupling. It actually is proportional to $(q-k)^\eta$ but the q^η part does not contribute to $\delta\mathcal{M}$. Similarly, the $KK\gamma$ coupling is proportional to $(2q-k)_\mu$, but the k_μ part is omitted in Eq. (5.58), since it does not contribute to $\delta\mathcal{M}$. Finally, the part proportional to $v^\alpha v^\eta$ in the D_s^* propagator also does not contribute to $\delta\mathcal{M}$ and is not displayed in Eq. (5.58).

Combining denominators using Eq. (3.6) gives

$$\begin{aligned} \delta\mathcal{M} &= \frac{16ig_\pi^2 e}{f^2} \epsilon_{\alpha\lambda\beta\nu} v^\lambda k^\alpha \epsilon^\beta (D_s^*) \epsilon_\mu^* (\gamma) \\ &\quad \times \int_0^\infty d\lambda \int \frac{d^n q}{(2\pi)^n} \frac{q^\nu q^\mu}{\left(q^2 + 2\lambda v \cdot q - m_K^2\right)^3}. \end{aligned} \tag{5.59}$$

Shifting the integration variable q by λv, we find this becomes

$$\delta\mathcal{M} = \frac{16ig_\pi^2 e}{nf^2}\epsilon_{\alpha\lambda\beta\mu} v^\lambda k^\alpha \epsilon^\beta(D_s^*)\epsilon^{*\mu}(\gamma)\int_0^\infty d\lambda \int \frac{d^n q}{(2\pi)^n}\frac{q^2}{\left(q^2 - m_K^2 - \lambda^2\right)^3}. \tag{5.60}$$

Consequently, the contribution of this Feynman diagram to the transition magnetic moment is

$$\delta\mu_3^{(\ell)} = \frac{16ig_\pi^2}{nf^2}\int_0^\infty d\lambda \int \frac{d^n q}{(2\pi)^n}\frac{q^2}{\left(q^2 - m_K^2 - \lambda^2\right)^3}. \tag{5.61}$$

Performing the q integration using Eq. (1.44) yields

$$\delta\mu_3^{(\ell)} = -\frac{4g_\pi^2\Gamma(2-n/2)}{f^2 2^n \pi^{n/2}}\int_0^\infty d\lambda\left(\lambda^2 + m_K^2\right)^{-2+n/2}. \tag{5.62}$$

Using Eq. (3.11), we find it easy to see that the integral over λ is proportional to $\epsilon = 4 - n$ and so the expression for $\delta\mu_3^{(\ell)}$ is finite as $\epsilon \to 0$. Taking this limit, we find

$$\delta\mu_3^{(\ell)} = \frac{g_\pi^2 m_K^2}{2\pi^2 f^2}\int_0^\infty \frac{d\lambda}{\left(\lambda^2 + m_K^2\right)} = \frac{g_\pi^2 m_K}{4\pi f^2}. \tag{5.63}$$

A similar calculation can be done for the other diagrams. Identifying f with f_K for the kaon loops, and with f_π for the pion loops, we have

$$\begin{aligned}
\mu_1^{(\ell)} &= \frac{2}{3}\beta - \frac{g_\pi^2 m_K}{4\pi f_K^2} - \frac{g_\pi^2 m_\pi}{4\pi f_\pi^2},\\
\mu_2^{(\ell)} &= -\frac{1}{3}\beta + \frac{g_\pi^2 m_\pi}{4\pi f_\pi^2},\\
\mu_3^{(\ell)} &= -\frac{1}{3}\beta + \frac{g_\pi^2 m_K}{4\pi f_K^2}.
\end{aligned} \tag{5.64}$$

Using f_K for kaon loops and f_π for pion loops reduces somewhat the magnitude of the kaon loops compared with the pion loops. Experience with kaon loops in chiral perturbation theory for interactions of the pseudo-Goldstone bosons with nucleons suggests that such a suppression is present.

It remains to consider the isospin violating decay $D_s^{*+} \to D_s^+\pi^0$. The two sources of isospin violation are electromagnetic interactions and the difference between the d and u quark masses, $m_d - m_u$. In chiral perturbation theory the pole type diagram in Fig. 5.3 dominates the part of the amplitude coming from the quark mass difference. The $\eta - \pi^0$ mixing is given in Eq. (1.104). Using this

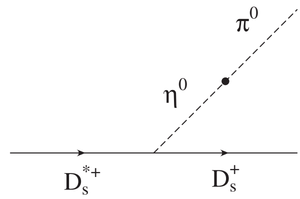

Fig. 5.3. Leading contribution to the isospin violating decay $D_s^* \to D_s \pi^0$.

and Eq. (5.20), we find the decay rate is

$$\Gamma\left(D_s^{*+} \to D_s^+ \pi^0\right) = \frac{g_\pi^2}{48\pi f^2} \left[\frac{m_d - m_u}{m_s - (m_u + m_d)/2}\right]^2 |\mathbf{p}_\pi|^3. \tag{5.65}$$

The measured mass difference $m_{D_s^*} - m_{D_s} = 144.22 \pm 0.60$ MeV implies that $|\mathbf{p}_\pi| \simeq 49.0$ MeV. In chiral perturbation theory, this is the dominant contribution coming from the quark mass difference because it is suppressed by only $(m_d - m_u)/m_s \simeq 1/43.7$, as opposed to $(m_d - m_u)/4\pi f$. The isospin violating electromagnetic contribution is expected to be less important since α/π is smaller than $(m_d - m_u)/m_s$.

The measured branching ratios in Table 5.1 determine the values of g_π and β. There are two solutions, since one has to solve a quadratic equation. Using the above results gives either ($g_\pi = 0.56$, $\beta = 3.5\ \text{GeV}^{-1}$) or ($g_\pi = 0.24$, $\beta = 0.85\ \text{GeV}^{-1}$). In evaluating these parameters, we have set $f = f_\pi$ for the hadronic modes. The values obtained for g_π are smaller than the quark model prediction discussed in Sec. 5.2. Of course there is a large uncertainty in this determination of g_π, since the experimental errors on branching ratios for the isospin violating decay $D_s^{*+} \to D_s^+ \pi^0$ and the radiative decay $D^{*+} \to D^+ \gamma$ are large, and because higher-order terms in chiral perturbation theory that have been neglected may be important.

5.5 Chiral corrections to $\bar{B} \to D^{(*)} e \bar{\nu}_e$ form factors

In Chapter 4, the nonperturbative order Λ_{QCD}/m_Q, corrections to B decay form factors, such as the semileptonic $\bar{B} \to D^{(*)} e \bar{\nu}_e$ form factors, $h_\pm(w)$, $h_V(w)$ and $h_{A_j}(w)$, were discussed. It seems reasonable that nonperturbative corrections to the form factors should be expandable in powers of $(\Lambda_{\text{QCD}}/m_Q)$, since the Lagrangian has an expansion in inverse powers of the heavy quark mass m_Q. However, because of the small values for the u and d quark masses, this ends up not being the case because of pole and loop diagrams involving pions. This point is illustrated below with the help of two examples: $\bar{B} \to D^* \pi e \bar{\nu}_e$, which has pole terms, and $\bar{B} \to D e \bar{\nu}_e$, which has pion loop terms.

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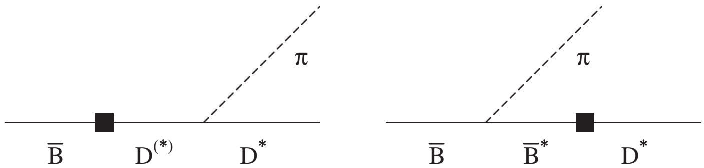

Fig. 5.4. Pole diagram contribution to the $\bar{B} \to D^*\pi e\bar{\nu}_e$ form factors. The solid box is an insertion of the axial current, Eq. (5.66).

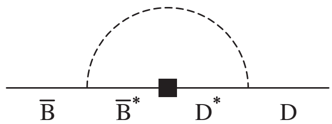

Fig. 5.5. One-loop correction to the $\bar{B} \to De\bar{\nu}_e$ form factors.

The weak current $\bar{c}\gamma_\mu(1-\gamma_5)b$ is a singlet under chiral $SU(3)_L \times SU(3)_R$ transformations. At leading order in chiral perturbation theory, this operator is represented in the chiral Lagrangian by

$$\bar{c}\gamma_\mu(1-\gamma_5)b = -\xi(w)\text{Tr}\,\bar{H}^{(c)}_{av'}\gamma_\mu(1-\gamma_5)H^{(b)}_{av}, \tag{5.66}$$

where we have now put back the heavy quark and velocity labels. $\xi(w)$ is the Isgur-Wise function.

Equation (5.66) contains no powers of the pion fields. This implies that at leading order in chiral perturbation theory the $\bar{B} \to D^*\pi e\bar{\nu}_e$ amplitudes come from pole diagrams in Fig. 5.4. The propagator for the intermediate D meson is

$$\frac{i}{p_\pi \cdot v + \Delta^{(c)}} \tag{5.67}$$

where p_π is the pion momentum and $\Delta^{(c)}$ is the D^*–D mass difference, which is of the order of $\Lambda^2_{\text{QCD}}/m_c$. Clearly, the form factors for this decay depend on $\Delta^{(c)}/v \cdot p_\pi$, and so do not simply have an expansion in Λ_{QCD}/m_Q. A similar conclusion holds for the $\bar{B} \to \pi e\bar{\nu}_e$ form factors discussed in Sec. 5.3.

To compute $\bar{B} \to De\bar{\nu}_e$ form factors, one needs the $\bar{B} \to D$ matrix element of Eq. (5.66). The leading order of chiral perturbation theory is the tree-level matrix element of this operator. At higher order in chiral perturbation theory one needs loop diagrams, as well as additional terms in Eq. (5.66) involving derivatives and insertions of the light quark mass matrix. At one loop the diagram in Fig. 5.5 contributes to the form factors for $\bar{B} \to De\bar{\nu}_e$ decay. This contribution is proportional to $g^2_\pi/(4\pi f)^2$ and depends also on the pion mass, m_π, and the $D^* - D$ mass difference, $\Delta^{(c)}$ (here, for simplicity, we neglect the $B^* - B$ mass difference). At zero recoil, Fig. 5.5, the wave-function renormalization diagrams

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and a tree-level contribution from an order $1/m_c^2$ operator give the contribution

$$\delta h_+(1) = -\frac{3g_\pi^2}{32\pi^2 f^2}\Delta^{(c)2}\left\{\ln\frac{\mu^2}{m_\pi^2} + F\left[\Delta^{(c)}/m_\pi\right] + C\right\}, \tag{5.68}$$

where μ is the scale parameter of dimensional regularization, and F is a dimensionless function that can be computed by explicitly evaluating the diagrams. Here C is the contribution of the local order $1/m_c^2$ operator. Any dependence on μ in a Feynman diagram is logarithmic. The mass difference $\Delta^{(c)}$ is of the order of $1/m_c$, and the pion mass is of the order of $\sqrt{m_q}$. Expanding F in a power series in $\Delta^{(c)}$ is equivalent to an expansion in powers of $1/m_c$. Expanding in powers of $\Delta^{(c)}$ gives

$$F = -\frac{3\pi}{4}\frac{\Delta^{(c)}}{m_\pi} + \frac{6}{5}\frac{\Delta^{(c)2}}{m_\pi^2} + \cdots. \tag{5.69}$$

Dimensional analysis dictates that for terms in $\delta h_+(1)$ of the order of $[\Delta^{(c)}]^n \sim (1/m_c)^n$, $n = 3, 4 \ldots$, the coefficients have the form $1/m_\pi^{n-2}$ and diverge as $m_\pi \to 0$. Nonperturbative corrections to the form factor $h_+(1)$ are not suppressed by powers of $(\Lambda_{\rm QCD}/m_c)$ but are much larger, of the order of $\Lambda_{\rm QCD}^{3n/2+2}/m_c^{n+2}\, {m_q}^{n/2}$ for $n \geq 0$. Note that in accordance with Luke's theorem there is no order $1/m_c$ term in $\delta h_+(1)$.

The heavy quark limit is m_c large, and the chiral limit is m_q small. Expanding F in powers of $\Delta^{(c)}$ is equivalent to taking the heavy quark limit where m_c is large while keeping m_q fixed. If one first takes the chiral limit where m_q is small while keeping m_c fixed, one should instead expand in powers of m_π. This expansion has the form

$$F = \left[\frac{2}{3} - \ln\frac{4\Delta^{(c)2}}{m_\pi^2}\right] + \frac{m_\pi^2}{\Delta^{(c)2}}\left[\frac{9}{2} - \frac{3}{2}\ln\frac{4\Delta^{(c)2}}{m_\pi^2}\right] - 2\pi\frac{m_\pi^3}{\Delta^{(c)3}} + \cdots, \tag{5.70}$$

with coefficients with positive powers of m_c in the higher order terms, which diverge as $m_c \to \infty$.

While the expansions in Eqs. (5.69) and (5.70) have divergent coefficients in the $m_\pi \to 0$ and $m_c \to \infty$ limits, respectively, the contribution of Fig. 5.5 and the wave-function renormalization diagrams to $h_+(1)$ is perfectly well defined. The chiral Lagrangian for heavy mesons can be used as long as $\Delta^{(c)}$ and m_π are both much smaller than $\Lambda_{\rm CSB}$, and the correction Eq. (5.68) is smaller than unity, irrespective of the value for the ratio $\Delta^{(c)}/m_\pi$. The interesting terms which cause divergent coefficients arise because of the ratio of two small scales, m_π and $\Delta^{(c)}$, and all such effects are computable in chiral perturbation theory.

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5.6 Problems

1. Compute the magnetic moments of the baryon octet in the nonrelativistic constituent quark model, and compare the results with the experimental data.

2. Neglecting the u and d quark masses, show that in chiral perturbation theory

$$\frac{f_{B_s}}{f_B} = 1 - \frac{5}{6}\left(1 + 3g_\pi^2\right)\frac{m_K^2}{16\pi^2 f^2}\left(\ln\frac{m_K^2}{\mu^2} + C\right) + \cdots .$$

C is a constant and the ellipsis denotes terms of higher order in chiral perturbation theory. The $\ln(m_K^2/\mu^2)$ term is referred to as a "chiral logarithm." The μ dependence of this term is canceled by a corresponding μ dependence in the coefficient C. If m_K were extremely small, the logarithm would dominate over the constant C.

3. The form factors for $D \to K\pi\bar{e}\nu_e$ are defined by

$$\begin{aligned}\langle\pi(p_\pi)K(p_K)|\bar{s}\gamma_\mu P_L c|D(p_D)\rangle = {} & i\omega_+ P_\mu + i\omega_- Q_\mu \\ & + ir(p_D - P)_\mu + h\epsilon_{\mu\alpha\beta\gamma}p_D^\alpha P^\beta Q^\gamma ,\end{aligned}$$

where

$$P = p_K + p_\pi , \qquad Q = p_K - p_\pi .$$

Use chiral perturbation theory to express the form factors $\omega_\pm$, r and h for $D^+ \to K^-\pi^+\bar{e}\nu_e$ in terms of f_D, f, g_π, $\Delta^{(c)} = m_{D^*} - m_D$ and $\mu_s = m_{D_s} - m_D$.

4. Verify Eqs. (5.64) and (5.65).

5. Evaluate $F(\Delta/m_\pi)$ in Eq. (5.68). Expand in $1/m_c$ and m_π and verify Eqs. (5.69) and (5.70).

6. The low-lying baryons containing a heavy quark Q transforms as a **6** and $\bar{\mathbf{3}}$ under $SU(3)_V$. Under the full chiral $SU(3)_L \times SU(3)_R$ the fields that destroy these baryons transform as

$$S^\mu_{ab} \to U_{ac}U_{bd}S^\mu_{cd}, \qquad T_a \to T_a U^\dagger_{ab},$$

where (see Problem 10 in Chapter 2)

$$S^\mu_{ab} = \frac{1}{\sqrt{3}}(\gamma_\mu + v_\mu)\gamma_5 B_{ab} + B^{*\mu}_{ab} .$$

Velocity and heavy quark labels are suppressed here.

(a) In the case $Q = c$ identify the various components of the fields T_a, B_{ab}, and $B^{*\mu}_{ab}$ with baryon states in Table 2.1.

(b) Argue that at leading order in $1/m_Q$, m_q, and derivatives, the chiral Lagrangian for heavy baryon pseudo-Goldstone boson interactions is

$$\begin{aligned}\mathcal{L} = {} & -i\bar{S}^\mu_{ab}(v\cdot D)S_{\mu ab} + \Delta M\bar{S}^\mu_{ab}S_{\mu ab} + i\bar{T}_a(v\cdot D)T_a + ig_2\epsilon_{\mu\nu\sigma\lambda}\bar{S}^\mu_{ab}v^\nu S^\lambda_{cb}\mathbb{A}^\sigma_{ac} \\ & + g_3\left(\epsilon_{abc}\bar{T}_a S_{\mu cd}\mathbb{A}^\mu_{bd} + \text{h.c.}\right) .\end{aligned}$$

Define how the covariant derivative D acts on S^μ_{ab} and T_a.

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