

# ON THE ERROR EQUATION FOR DETERMINING EARTH ROTATION PARAMETERS WITH VLBI NETWORK

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## 1. INTRODUCTION

In this paper, the partial derivative matrix in the error equation for determining earth rotation parameters (ERP) with the VLBI network is described, and the effect of the functional parameter constraint on solving ERP is discussed.

## 2. THE COMPUTATION OF THE PARTIAL DERIVATIVE MATRIX FOR DELAY AND DELAY RATE WITH RESPECT TO PARAMETERS

The delay for computing partial derivatives may be written as

$$\tau = \tau_o [1 - (\dot{\mathbf{r}}_2)_s \cdot \mathbf{S} + ((\dot{\mathbf{r}}_2)_s \cdot \mathbf{S})^2] \quad , \quad (1)$$

where

$$\tau_o = -\frac{1}{c} \mathbf{B}_s \cdot \mathbf{S} = -\frac{1}{c} PN'S'WB_b \cdot \mathbf{S} \quad . \quad (2)$$

The following expression of the delay in the celestial coordinate system is given:

$$\tau = -\frac{\mathbf{B}_s}{c} [\sin \delta_B \sin \delta + \cos \delta_B \cos \delta \cos(\alpha_B - \alpha)] \quad . \quad (3)$$

Obviously,  $\alpha_B$  and  $\delta_B$  are the right ascension and declination of the baseline. If the Celestial Ephemeris Pole is adopted as the uniform pole for the space-fixed and body-fixed coordinate system, then  $\alpha_B = S_G + \lambda_B$ .

If we directly utilize equation (3) to compute the partial derivatives of the delay and delay rate with respect to the parameters and mix up the corresponding reference system, then the correct ERP cannot be calculated. The expressions of the delay and delay rate for computing partial derivatives should be in explicit function form of source position parameters in the space-fixed coordinate system and baseline parameters in the body-fixed coordinate system.

## 3. SOLVING FOR THE ERROR EQUATION

In determining the ERP, the radio source is observed simultaneously at all stations of the VLBI network. At that time, the constraint condition can sometimes be imposed in least squares adjustment.

The error equation of determining ERP with VLBI network is

$$\mathbf{L} = \mathbf{F} \Delta \mathbf{X} + \mathbf{e} \quad , \quad (4)$$

where  $\mathbf{L}$  is the residual vector of the delay and delay rate,  $\mathbf{F}$  is the partial derivative matrix of the delay and delay rate with respect to the parameters estimated,  $\Delta \mathbf{X}$  is the corrected vector of the *a priori* parameters, and  $\mathbf{e}$  is the vector of observational error.

By means of functional parameter constraints, we can derive the solution of  $\Delta \mathbf{X}$  (Mikhail 1970, 1976)

$$\Delta \mathbf{X}_{lsr} = \Delta \mathbf{X}_{ls} + (\mathbf{Z}^T - \Delta \mathbf{X}_{ls}^T \mathbf{Q}^T) (\mathbf{Q} \mathbf{D} \mathbf{Q}^T)^{-1} \mathbf{Q} \mathbf{D} \quad , \quad (5)$$

where  $\mathbf{Q}$  is the coefficient matrix of  $\Delta \mathbf{X}$  under the constraint condition, and  $\mathbf{Z}$  is the conditional vector constrained.

The variance covariance matrix for parameters estimated is

$$\mathbf{D}_R = \mathbf{D} - \mathbf{D} \mathbf{Q}^T (\mathbf{Q} \mathbf{D} \mathbf{Q}^T)^{-1} \mathbf{Q} \mathbf{D} \quad . \quad (6)$$

If the VLBI network is composed of three stations, then the general constraint condition is (Davidson 1980)

$$\sum_{m=1}^3 B_m = 0 \quad , \quad \sum_{m=1}^3 a_m^k = 0 \quad , \quad (7)$$

where  $m$  is the baseline number, and  $k$  is the order of clock polynomial coefficients.

In order to examine the effect of the functional parameter constraint on the precision of earth rotation parameters estimated, we take three baselines composed of three stations: Shanghai (China), Effelsberg (FRG), and Algonquin (Canada). The positions of 10 radio sources whose correlation flux densities are greater than 0.5 Jy are taken from the catalogue given by Preston *et al.* (1978).

The altitude  $h$  and the azimuth  $A$  of these radio sources observed from the station are computed by means of the positions of the stations and the radio sources, once every 15 minutes of Greenwich sidereal time. The observed altitude is greater than 20°. Therefore the observational schedule can be made.

The clock model takes a linear form. The standard deviations,  $\sigma_r$  and  $\sigma_{\dot{r}}$ , of the delay and delay rate observed, are assumed to be equal to 0.5 ns and 0.1 ps s<sup>-1</sup>, respectively.

The computation results show that the functional parameter constraint imposed in the least squares adjustment may reduce the correlation of ERP with the baseline parameters and clock polynomial coefficients. All of these parameters can be solved together, and the precision of solving ERP can be increased if the functional parameter constraint is used.

#### 4. REFERENCES

- Davidson, D. A. 1980, *A Least Square Adjustment for LBI*, Univ. of New Brunswick, Technical Report No. 71.  
 Mikhail, E. M. 1970, *Photogrammetric Engineering*, **36**, 1277.  
 Mikhail, E. M. 1976, *Observations and Least Squares* (New York: IEP).  
 Preston, R. A., *et al.* 1978, DSN Progress Report 42-46.