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TIME DELAY EFFECTS FOR MEASURING COSMOLOGICAL DISTANCES

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Lorsqu'une galaxie compacte et massive se situe entre un observateur O et une source de lumière S, il se peut que par déflection gravitationnelle, la lumière suive différents trajets de durées différentes pour parvenir à l'observateur. Si la source est variable il est possible de mesurer la différence de temps Δt entre les durées des trajets. Cela nous donne la possibilité de déterminer les distances cosmologiques d'une façon purement géométrique. Les problèmes liés à la distribution inconnue des masses et les possibilités observationnelles sont discutés.

We consider a light source S (for instance a QSO) which is lying behind and close to the line of sight of a distant massive galaxy B. The light from S to the observer O can then, due to the gravitational deflection of light follow two different paths, 1 and 2, as indicated on Fig. 1, see Refsdal (1964a). B which has a mass M is assumed to be spherically symmetric. According to Einstein's theory of relativity, the deflection of a ray of light when passing B at a distance r is

$$v = 4GM/c^2 r \tag{1}$$

The angle α between the two images S_1 and S_2 is

$$\alpha = \sqrt{\alpha_o^2 + \beta^2} \tag{2}$$

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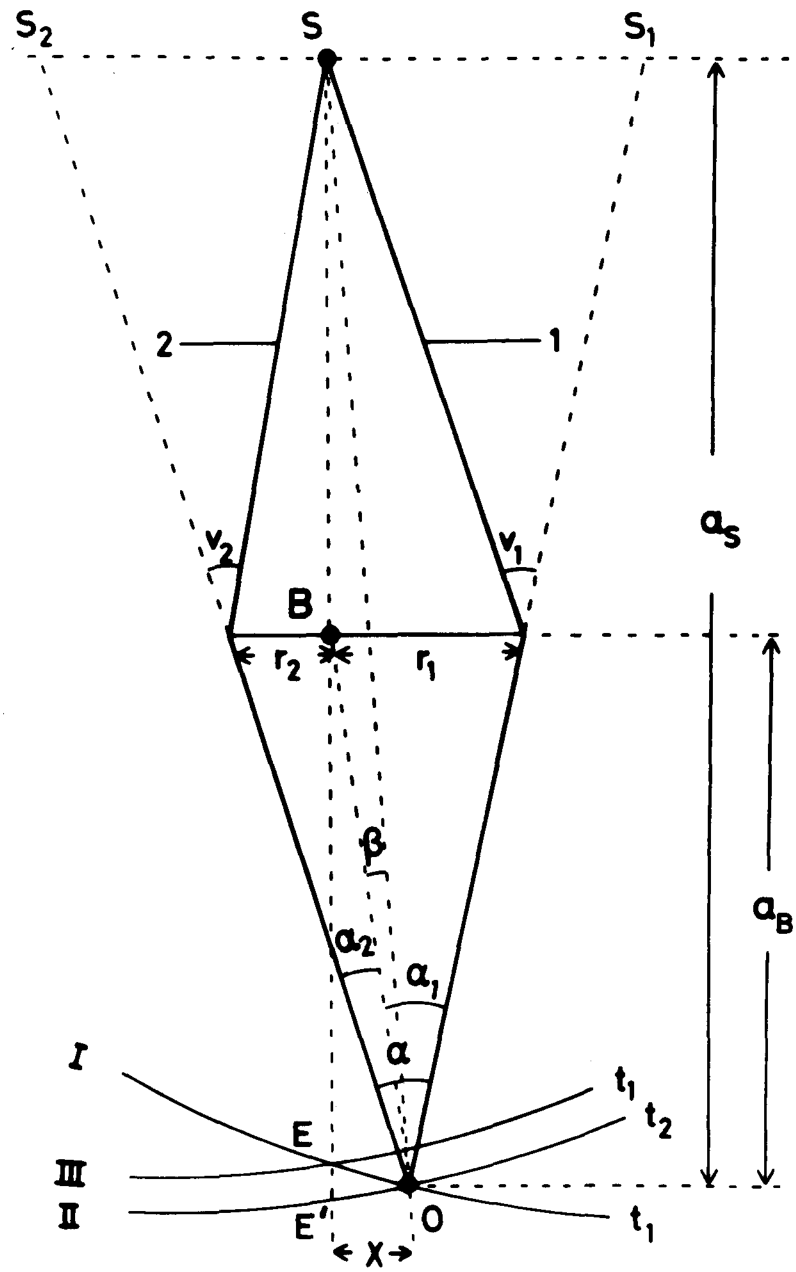

The two light rays from S to O and the wavefronts I, II and III

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where

$$\alpha_o = \frac{4}{c}\sqrt{\frac{GM}{n a_B}} \qquad (3)$$

and

$$n = a_S / (a_S - a_B) \qquad (4)$$

Here a_S and a_B are the distances to S and B, respectively, and β is the angular separation between S and B in the absence of any deflection. For values of $a_B = 10^9$ pc, $M = 10^{12} M_{\odot}$ and $n = 1$ we get $\alpha_o = 5.7''$. The apparent luminosity of S_1 and S_2 are (Refsdal 1964a).

$$L_1 = \frac{1}{4}\left(2 + \frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) L_N \qquad (5)$$

and

$$L_2 = \frac{1}{4}\left(-2 + \frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) L_N \qquad (6)$$

where L_N is the normal apparent luminosity of S (without lens effect).

The light travel times for the two paths can differ by up to one year or more and this difference Δt can be measured if S is varying, see Refsdal (1964b), hereafter referred to as Paper I. There has been some confusion as to how Δt was calculated in Paper I, and we therefore briefly discuss this point. We consider then the wavefronts which are drawn in Fig. 1. At the observer the two wavefronts I and II with light travel times t_1 and t_2, respectively, are indicated. Wavefront III crosses wavefront I at E which lies on the extension of the line SB. Because of symmetry the light travel time from S to E must be the same for all rays reaching E and wavefront III has therefore also light travel time t_1. The distance between wavefronts I and III (distance E E') is therefore equal to $c\ (t_2 - t_1) = c\ \Delta t$. Since $\beta < 0.4\ \alpha_o$ in cases of practical interest (otherwise $L_2 < 0.27\ L_N$, see Eq (6)) we neglect terms of order β^2 / α_o^2 and see then from Fig. 1 ($\angle EDE' = \alpha$)

$$\Delta t = c^{-1} \alpha\ x \qquad (7)$$

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where x is the distance from O to E. By measuring α and Δt we can therefore determine x which plays the rôle of a baseline in this case. From Eqs. (3) and (7) we get

$$\Delta t = \frac{16\,G}{c^3} \cdot \frac{\beta}{\alpha_o} M = 2.5 \frac{\beta}{\alpha_o} \cdot \frac{M}{10^{12} M_\odot} \tag{8}$$

It is clear that the value of Δt derived here correctly takes into account the difference in the length of the light path and also the change in the photon-velocity due to the gravitational field of the deflector, see Cooke and Kantowski (1975). An expression for the Hubble constant H in terms of observable quantities can now easily be derived. (see Paper I).

$$H = \frac{Z_S Z_B \alpha (\alpha_1 - \alpha_2)}{\Delta t (Z_S - Z_B)} \tag{9}$$

Z_S and Z_B are the redshifts of S and B, respectively, and α_1 and α_2 the angles between S_1 and B and S_2 and B respectively. Since $(\alpha_1 - \alpha_2)$ could be pretty difficult to determine accurately, an alternative expression can be derived (see Paper I).

$$H = \frac{Z_S Z_B \alpha^2 (\sqrt{L_1 / L_2} - 1)}{\Delta t (Z_S - Z_B)(\sqrt{L_1 / L_2} + 1)} \tag{10}$$

When the redshifts are large a correction term depending on the cosmological model has to be included in Eqs. (9) and (10). For $Z_B \approx 1$ this correction can typically amount to $\pm$ 20 %, and this gives us a possibility of testing cosmological models, see Refsdal (1966).

We have till now assumed that $v \sim r^{-1}$ which is valid as long as axial symmetry is retained and the rays pass outside the deflecting mass. If only the first condition is fulfilled, the deflection is directed towards the symmetry axis, $v = v(r)$, and as a first approximation we can write

$$v \sim r^{\varepsilon - 1} \tag{11}$$

where ε is a parameter which will usually be between 0 and 1. Instead of Eqs. (9) and (10) we now get ($0 < \varepsilon < 1$).

$$H = \frac{Z_S Z_B \alpha (\alpha_1 - \alpha_2)}{t (Z_S - Z_B)} \cdot \frac{2 - \varepsilon}{2} \tag{12}$$

$$\approx \frac{Z_S Z_B \alpha^2}{\Delta t (Z_S - Z_B)} \cdot \frac{(L_1/L_2)^{0.5(1+\varepsilon)} - 1}{(L_1/L_2)^{0.5(1+\varepsilon)} + 1} \cdot \frac{2 - \varepsilon}{2}$$

For cases without axial symmetry a ray tracing method turns out to be more convenient than analytical methods, and Δt is most easily found by integrating along the light rays, see Cooke and Kantowski(1975). Application of Eqs. (9), (10) or (12) would now give incorrect values of H, and one must in some cases expect errors larger than a factor 2. This problem is presently being investigated in Hamburg. For large redshifts the effect of a lumpy universe and the empty light cone effect should also be investigated in this connection, see Zel'dovich(1964), Bertotti(1966), Gunn(1967), Kantowski(1969), Refsdal(1970) and Dyer and Roeder(1974).

The possibility of observing the effect depends on the number of suitable light sources and on the distribution and masses of suitable deflectors. Restricting ourselves to cases with $\beta < 0.4\alpha_o$ we get for each deflector an effective solid angle of $\Omega = 0.16\pi\alpha_o^2$ within which the background object must be located. From Eq. (3) we then find for n = 1 that $\Omega = 8GM/(c^2 a_B)$, i.e. is proportional to the gravitational potential Φ of the deflecting mass at the observer. By adding the Φ-values for all suitable deflectors out to a certain distance (or redshift Z_{max}) we find that the total effective solid angle is about $4\pi\, \delta_d Z^2_{max_2}$ ($Z_{max} < 0.5$) where δ_d is the density parameter ($\delta_d = 4\pi G\rho_d / 3H^2$) correspond-

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ing to the smeared-out density ρ_d of the suitable deflectors. Denoting the number of suitable background objects over the whole sky by N we find that the expected number of cases with $\beta < 0.4 \alpha_o$ is

$$P = N \delta_d Z^2_{max} = 350 Z^2_{max} \qquad (13)$$

We have here assumed that δ_d is one percent of the density parameter δ_L of luminous matter ($\delta_L = 0.035$), and that $N = 10^6$ (this is the number of QSO^s with $m_v < 21$ and $Z < 2.5$ according to the $10^{5\tau}$ density evolution law of Schmidt (1972). It can therefore not be ruled out that the effect should be possible to observe for relative small values of the deflector redshift ($Z_B < 0.1$). Since a search for the effect among 10^6 background objects would be extremely time consuming however, a simple method for identifying massive compact galaxies seems necessary in order to make the search reasonably effective.

Some possibly observed cases have already been reported in the literature, see Gott and Gunn (1974) and Sanitt (1976). These are however still in the speculative stage. We finally point out that the luminosity increases which would typically occur in the cases discussed here are very much smaller than the factor 50 or more which is needed to "explain" QSO^s as lens images of nuclei of Seyfert galaxies.

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DISCUSSION

K. RUDNICKI: I did not understand what kind of observations and what accuracy of them you do need to apply your method for estimation of distances?

S. REFSDAL: One must observe the quantities given on the right hand side of Eq(9) and /or Eq. (10). The main uncertainty will probably come from the factor $(\alpha_1 - \alpha_2)$ in Eq. (9) and L_1/L_2 in Eq. (10). In addition to the observational errors come the uncertainties due to the unknown mass distribution, which usually will be more important.

J.E. GUNN: Gott and I found in a similar study that the time delay depends crucially on the unknown density distribution of the deflector, so the test is not likely to be very useful. Also, the density required to make two images is higher than is likely to exist even in compact galaxies.

S. REFSDAL: It is true that the mass distribution in the deflector is very important. Since in the paper with Gott you did not take into account the slowing down of the photons close to the deflector you got wrong values of Δt, see Cooke and Kantowski (1975). The correct Δt depends less critically on the mass distribution than the Δt used in your paper, so the situation is somewhat better than your results indicate. Also, the fact that we have two different expressions for H makes it in principle possible to get some information on the mass distribution, and thereby reduce the uncertainty. The number density of galaxies which are massive enough and compact enough for our purpose is rather uncertain, but I do not think that the estimate $\sigma_d = 0.01\ \sigma_L$ can be ruled out at the moment.