

# Explaining the density profile of self-gravitating systems by statistical mechanics

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**Abstract.** A self-gravitating system usually shows a quasi-universal density profile, such as the NFW profile of a simulated dark matter halo, the flat rotation curve of a spiral galaxy, the Sérsic profile of an elliptical galaxy, the King profile of a globular cluster and the exponential law of the stellar disk. It will be interesting if all of the above can be obtained from first principles. Based on the original work of White & Narayan (1987), we propose that if the self-bounded system is divided into infinite infinitesimal subsystems, the entropy of each subsystem can be maximized, but the whole system's gravity may just play the role of the wall, which may not increase the whole system's entropy  $S_t$ , and finally  $S_t$  may be the minimum among all of the locally maximized entropies (He & Kang 2010). For spherical systems with isotropic velocity dispersion, the form of the equation of state will be a hybrid of isothermal and adiabatic (Kang & He 2011). Hence this density profile can be approximated by a truncated isothermal sphere, which means that the total mass must be finite and our results can be consistent with observations (Kang & He 2011b). Our method requires that the mass and energy should be conserved, so we only compare our results with simulations of mild relaxation (i.e. the virial ratio is close to -1) of dissipationless collapse (Kang 2014), and the fitting also is well. The capacity can be calculated and is found not to be always negative as in previous works, and combining with calculations of the second order variation of the entropy, we find that the thermodynamical stability still can be true (Kang 2012) if the temperature tends to be zero. However, the cusp in the center of dark matter halos can not be explained, and more works will continue.

The above work can be generalized to study the radial distribution of the disk (Kang 2015). The energy constraint automatically disappears in our variation, because angular momentum is much more important than energy for the disk-shape system. To simplify this issue, a toy model is taken: 2D gravity is adopted, then at large scale it will be consistent with a flat rotation curve; the bulge and the stellar disk are studied together. Then with constraints of mass and angular momentum, the calculated surface density can be consistent with the truncated, up-bended or standard exponential law. Therefore the radial distribution of the stellar disk may be determined by both the random and orbital motions of stars. In our fittings the central gravity is set to be nonzero to include the effect of asymmetric components.

**Keywords.** galaxies:general, galaxies:spiral, equation of state, method:n-body simulation

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