A NOTE ON ANNIHILATOR AND COMPLEMENTED BANACH ALGEBRAS

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1. Introduction

The purpose of this paper is to generalize some results in a recent paper by Tomiuk and the author ([11]).

In section 3, we study the Arens product, weakly completely continuous (w.c.c.) and annihilator Banach algebras. We show that if A is a semi-simple Banach algebra which is a dense two-sided ideal of a semi-simple annihilator Banach algebra B, then A is w.c.c. This result greatly generalizes [11, page 56, Theorem 6.1]. We also obtain that if A is a semi-simple Banach algebra which is a dense two-sided ideal of a B^*-algebra B, then A is an annihilator algebra if and only if A is w.c.c. and A^2 is dense in A. This is a generalization of [11, page 57, Theorem 6.2].

Section 4 is devoted to the study of complementors on Banach algebras. Let A be a semi-simple annihilator Banach algebra which is a dense subalgebra of a B^*-algebra B. We show that if $x \in \mathrm{cl}_A(xA)$ for all x in A and A is a two-sided ideal of B, then every complementor p on B induces a complementor q on A. Conversely, of $\| \cdot \|$ majorizes $| \cdot |$ on A and the constant M in (4.3) is not zero, then we get that every complementor q on A can be extended to a complementor p on B. These two results improve [11, page 60, Theorem 8.2] and [11, page 63, Theorem 8.8].

2. Notation and preliminaries

Definitions not explicitly given are taken from Rickart's book [9].

For any subset E of a Banach algebra A, let $l_A(E)$ and $r_A(E)$ denote the left and right annihilators of E in A, respectively. Then A is called a modular annihilator algebra if, for every maximal modular left ideal I and for every maximal modular right ideal L, we have $r_A(I) = (0)$ if and only if $I = A$ and $l_A(J) = (0)$ if and only if $J = A$.

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An element x in a Banach algebra A is said to be weakly completely continuous (w.c.c.) if the left and right multiplication operators of x are weakly completely continuous. It is known that the set of all w.c.c. elements of A forms a closed two-sided ideal in A (see [8]). If each element of A is w.c.c., then A is called w.c.c.

Let A be a Banach algebra, A^* and A^{**} the conjugate and second conjugate spaces of A, respectively. The mapping π_A will denote the canonical mapping of A into A^{**}. The Arens product on A^{**} is defined in stages according to the following rules (see [2]). Let $x, y \in A$, $f \in A^*$ and $F, G \in A^{**}$.

(a) Define $f \circ x$ by $(f \circ x)(y) = f(xy)$. Then $f \circ x \in A^*$.

(b) Define $G \circ f$ by $(G \circ f)(x) = G(f \circ x)$. Then $G \circ f \in A^*$.

(c) Define $F \circ G$ by $(F \circ G)(f) = F(G \circ f)$. Then $F \,_\gamma\, G \in A^{**}$.

A^{**} with the Arens product $\circ$ is denoted by $(A^{**}, \circ)$.

Let A be a Banach algebra and let L_r be the set of all closed right ideals of A. Following [10], we shall say that A is a (right) complemented Banach algebra if there exists a mapping $p: R \to R^p$ of L_r into itself having the following properties:

(c_1) $R \cap R^p = (0) \quad (R \in L_r)$;

(c_2) $R + R^p = A \quad (R \in L_r)$;

(c_3) $(R^p)^p = R \quad (R \in L_r)$;

(c_4) if $R_1 \supset R_2$, then $R_2^p \supset R_1^p (R_1, R_2 \in L_r)$.

The mapping p is called a (right) complementor on A.

Let A be a Banach algebra which is a subalgebra of a Banach algebra B. For each subset E of A, cl(E) (resp. cl$_A(E)$) will denote the closure of E in B (resp. A). $l(E)$ (resp. $l_A(E)$) will denote the left annihilator of E in B (resp. A). We write $\| \cdot \|$ for the norm on A and $| \cdot |$ for the norm on B.

An idempotent e in a Banach algebra A is said to be minimal if eAe is a division algebra. In case A is semi-simple, this is equivalent to saying that $Ae(eA)$ is a minimal left (right) ideal of A.

In this paper, all algebras and linear spaces under consideration are over the field C of complex numbers.

3. The Arens product and w.c.c. algebras

Let A be a semi-simple Banach algebra which is a dense two-sided ideal of a semi-simple Banach algebra B, Then by [5, page 3, Proposition 2.2], there exists a constant $K > 0$ such that

$$K \| a \| \geqq | a | \qquad (a \in A) \tag{3.1}$$

and hence by [5, p. 3, Theorem 2.3], there exists a constant D such that

$$\| ab \| \leqq D \| a \| \; | b | \quad \text{and} \quad \| ba \| \leqq D \| a \| \; | b | \tag{3.2}$$

for all a in A and b in B.

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LEMMA 3.1. *Let A be a semi-simple Baanch algebra which is a dense two-sided ideal of a semi-simple annihilator Banach algebra B. Then $\pi_A(A)$ is a two-sided ideal of $(A^{**}, \circ)$.*

PROOF. This is Theorem 3.1 in [15].

LEMMA 3.2. *Let A be a Banach algebra, Then $\pi_A(A)$ is a two-sided ideal of $(A^{**}, \circ)$ if and only if A is wcc.*

PROOF. Suppose that $\pi_A(A)$ is a two-sided ideal of A^{**}. Let $\{x_\alpha\}$ be a bounded net in A. Then by Alaoglu's Theorem [6, page 424, Theorem 2], we can assume that $\pi_A(x_\alpha) \to F$ weakly for some F in A^{**}. Since $\pi_A(x) \circ F \in \pi_A(A)$ for all x in A, it follows that $\pi_A(xx_\alpha) \to \pi_A(x) \circ F$ weakly in $\pi_A(A)$. Hence A is w.c.c.

Conversely suppose that A is w.c.c. Let $x \in A$ and $F \in A^{**}$ with $\|F\| = 1$. By Goldstine's Theorem [6, page 424, Theorem 5], there exists a net $\{x_\alpha\} \subset A$ with $\|x_\alpha\| \leqq 1$ such that $\pi_A(x_\alpha) \to F$ weakly. Since x is w.c.c., we can assume that $xx_\alpha \to y$ weakly for some y in A. It follows that $\pi_A(x) \circ F = \pi_A(y) \in \pi_A(A)$. Consequently $\pi_A(A)$ is a two-sided ideal of A^{**} and this completes the proof.

REMARK. Lemma 3.2 was obtained for B^*-algebra in the proof of [7, page 84, Theorem].

THEOREM 3.3. *Let A be a semi-simple Banach algebra which is a dense two-sided ideal of a semi-simple annihilator Banach algebra B. Then A is w.c.c. and, in particular, B is w.c.c.*

PROOF. The is follows from Lemmas 3.1 and 3.2.

Theorem 3.3 is a generalization of [11, page 56, Theorem 6.1].

THEOREM 3.4. *Let A be a semi-simple Banach algebra which is a dense two-sided ideal of a B^*-algebra B. Then the following statements are equivalent.*

(i) *A is a modular annihilator algebra.*

(ii) *$\pi_A(A)$ is a two-sided ideal of $(A^{**}, \circ)$.*

PROOF. (i) $\Rightarrow$ (ii). Suppose that A is a modular annihilator algebra. Let x be a positive element in B. Then $x = h^2$, where h is a hermitian element in B. Let $h_n \in A$ be such that $h_n \to h$ in $|\cdot|$. Then $h_n h_n^* \to h^2 = x$ in $|\cdot|$; clearly $h_n h_n^* \in A$. Now by the proof of [4, page 287, Lemma 2.6], the socle of B is dense in B, because any element of B is a linear combination of positive elements. Hence by [17, page 41, Lemma 3.11] and [17, page 42, Theorem 4.1], B is a dual B^*-algebra. Therefore $\pi_A(A)$ is a two-sided ideal of $(A^{**}, \circ)$ by Lemma 3.1 and this proves (ii).

(ii) $\Rightarrow$ (i). Suppose (ii) holds. Then by Lemma 3.2, A is w.c.c. Since $B^2 = B$, by (3.1), (3.2) and the proof of [8, page 29, Lemma 9], B is w.c.c. and so is dual.

([8, page 21, Theorem 6]). Therefore by [17, page 40, Theorem 3.7], A is a modular annihilator algebra. This completes the proof.

Theorem 3.4 generalizes [13, page 830, Theorem 5.2] and [14, page 112, Theorem 2.2].

THEOREM 3.5. *Let A be a semi-simple Banach algebra which is a dense two-sided ideal of a B^*-algebra B. Then A is an annihilator algebra if and only if A is w.c.c. and A^2 is dense in A.*

PROOF. Suppose A is w.c.c. and A^2 is sense in A. Then by the proof of Theorem 3.4, B is a dual algebra and A is a modular annihilator algebra. Let e be a minimal idempotent of B. Since $eAe = eBe = Ce$, it is easy to see that $e \in A$ and consequently, A and B have the same socle S. Since $Be = Ae$, it follows easily from (3.1) and the Closed Graph Theorem that $\| \cdot \|$ and $| \cdot |$ are equivalent on Ae. Let x and y be elements in A. Since S is dense in B, there exists a sequence $\{x_n\}$ in S such that $x_n \to x$ in $|\cdot|$. It follows from (3.2) that $x_n y \to xy$ in $\| \cdot \|$. Therefore $A^2 \subset \mathrm{cl}_A(S)$. Since A^2 is dense in A, $\mathrm{cl}_A(S) = A$.

Let I be a minimal closed two-sided ideal of A and $J = \mathrm{cl}(I)$. By [17, page 37, Lemma 3.1], I contains a minimal idempotent e. Therefore $I = \mathrm{cl}(AeA)$ and consequently $J = \mathrm{cl}(BeB)$. Hence J is a minimal closed two-sided ideal of B. By [9, page 100, Theorem (2.18.14)] and [9, 249, Theorem (4.9.2)], J is a simple dual B^*-algebra. Clearly I is a dense two-sided ideal of J and they have the same socle. Let L be a minimal left ideal of I. Then L is a minimal left ideal of J. By [9, page 261, Theorem (4.10.3)] and [9, page 263, Theorem (4.10.(9)] L is a Hilbert space in $| \cdot |$. It is well known that J can be considered as the algebra of all completely continuous linear operators on L. Since I is a dense two-sided ideal of J and they have the same socle, I contains all operators of finite rank on L. Since $\| \cdot \|$ and $| \cdot |$ are equivalent on L, it follows easily from [9, page 104, Theorem (2.8.23)] that I is an annihilator algebra. Therefore by [17, page 42, Theorem 3.12] and [9, page 106, Theorem (2.8.29)], A is an annihilator algebra. The converse of the theorem follows immediately from Theorem 3.3 and [9, page 100, Cotrollary (2.8.16)].

Theorem 3.5 is a generalization of [11, page 57, Theorem 6.2]. Also see [3, page 9, Theorem 3.5(2)].

LEMMA 3.6. *Let e be an idempotent is a semi-simple Banach algebra A. Then e is a w.c.c. element if and only if eA and Ae are reflexive.*

PROOF. Let $(eA)^{**}$ have the Arens product $\circ$ and let π be the canonical mapping of eA into $(eA)^{**}$. Suppose e is a w.c.c. element in A. Let $F \in (eA)^{**}$ with $\| F \| = 1$. Then by Goldstine's Theorem [6, page 424, Theorem 5], there exists a net $\{x_\alpha\}$ in eA such that $\| x_\alpha \| \leqq 1$ for all α and $\pi(x_\alpha) \to F$ weakly in $(eA)^{**}$. Therefore $\pi(x_\alpha) = \pi(ex_\alpha) \to \pi(e) \circ F$ weakly in $(eA)^{**}$.

Hence $F = \pi(e) \circ F$. Since e is w.c.c., there exists a subset $\{x_{\alpha_k}\} \subset \{x_\alpha\}$ and $y \in A$ such that $ex_{\alpha_k} \to y$ weakly in A. Consequently $F = \pi(y) \in \pi(eA)$. Hence eA is reflexive. Similarly Ae is reflexive.

Conversely, suppose that eA and Ae are reflexive. Let $\{x_\alpha\}$ be a net in A with $\|x_\alpha\| \leqq 1$. Since $\|ex_\alpha\| \leqq \|e\|$, by [6, page 425, Theorem 7]. We can assume that there exists some y in eA such that $f(ex_\alpha) \to f(y)$ for all $f \in (eA)^*$. Therefore it follows that e is w.c.c. and this completes the proof.

THEOREM 3.7. *Let A be semi-simple complemented Banach $*$-algebra. Then A is w.c.c.*

PROOF. By [10, page 655, Lemma 5], the socle S of A is dense in A. Let e be a minimal idempotent of A. Then by [10, page 656, Theorem 5], Ae is a Hilbert space and so it is reflexive. By [11, page 51, Lemma 4.1] and the above argument, eA is also reflexive. Now it follows from Lemma 3.6 that e is w.c.c. Consequently S is w.c.c. and so is A.

4. Induced complementors

Let A be a Banach algebra with a complementor p. A minimal idempotent f in A is called a p-projection if $(fA)^p = (-f)A$. If A is a semi-simple annihilator complemented Banach algebra, then every non-zero right ideal contains a p-projection and A contains a maximal orthogonal family of p-projections (see [10, page 654]).

THEOREM 4.1. *Let A be a semi-simple annihilator Banach algebra in which $x \in \mathrm{cl}_A(xA)$ for all x in A. If A is a dense two-sided ideal of a B^*-algebra B, then for every complementor p on B, the mapping $q: I \to \mathrm{cl}(I)^p \cap A$ on the closed right ideals I of A is a complementor on A.*

PROOF. By the proof of Theorem 3.4, B is a dual algebra and A and B have the socle S. Let $\{f_\alpha : \alpha \in \nabla\}$ be a maximal orthogonal family of p-projections in B. Then $f_\alpha \in S \subset A$ for all $\alpha \in \nabla$. In order that q be a complementor on A, by Lemma 2.2 and Theorem 3.2 in [16], it is sufficient to show that $x = \sum_\alpha f_\alpha x$ in $\|\cdot\|$ for all x in A. Since p is a complementor on B, by Theorem 3.2 in [16], $b = \sum_\alpha f_\alpha b$ in $|\cdot|$ for all b in B. Let $x, y \in A$ and $\alpha_i \in \nabla$ $(i = 1, \cdots, n)$. Since by (3.2) $\|\sum_{i=1}^n f_{\alpha_i} xy\| \leqq D\,|\sum_{i=1}^n f_{\alpha_i} x|\,\|y\|$, it follows that $\sum_\alpha f_\alpha xy$ is summable in $\|\cdot\|$. Since $x \in \mathrm{cl}_A(xA)$, for any given $\varepsilon > 0$, there exists some z in A such that $\|x - xz\| < \varepsilon$. Therefore by (3.2)

$$\left\| \sum_{i=1}^n f_{\alpha_i} x \right\| \leqq D \left| \sum_{i=1}^n f_{\alpha_i} \right| \|x - xz\| + \left\| \sum_{i=1}^n f_{\alpha_i} xz \right\|. \tag{4.1}$$

Since B is a B^*-algebra, it follows from Corollary 3.5 in [16] and the proof of

[12, page 259, Theorem 4] that $\{|\sum_{i=1}^{n} f_{\alpha_i}| : \alpha_i \in \nabla\}$ is bounded. Since $\sum_\alpha f_\alpha xz)$ is summable in $\|\cdot\|$ and ε is arbitrary, by (4.1) $\sum_\alpha f_\alpha x$ is summable in $\|\cdot\|$. Since $x = \sum_\alpha f_\alpha zx$ in $|\cdot|$, by (3.1) we have $x = \sum_\alpha f_\alpha x$ in $\|\cdot\|$. This completes the proof.

REMARK 1. Theorem 4.1 is a generalization of [11, page 60, Theorem 8.2]. Some arguments in the proof of Theorem 4.1 are similar to those in the proof of [8, page 30, Theorem 16].

REMARK 2. If B is not a B^*-algebra, then Theorem 4.1 is not true. In fact, let G be an infinite compact group with the Haar measure and let A be the algebra of all continuous functions on G, normed by the maximum of the absolute value. It is well known that A is a dual A^*-algebra which is a dense two-sided ideal of $L_2(G)$. The mapping $p: R \to l(R)^*$ is a complementor on $L_2(G)$, but the mapping $q: I \to l_A(I)^*$ is not a complementor on A (see [15] and [16]).

REMARK 3. If A is not an ideal of B, then Theorem 4.1 is not true. In fact, let $A = L_1(G)$ and B the completion of A in an auxiliary norm. Then the mapping $q: I \to l_A(I)^*$ is not a complementor on A, but the mapping $p: R \to l(R)^*$ is a complementor on B (see [16]).

We establish a converse to Theorem 4.1. Although [11, page 63, Theorem 8.8] is a result in this direction, its condition is very restricted. We shall show that a similar result holds for a much larger class of algebras.

In the rest of this section, A will be a semi-simple annihilator Banach algebra such that $x \in \mathrm{cl}_A(xA)$ for all $x \in A$ which is a dense subalgebra of a B^*-algebra B. Suppose there exists a constant K such that $K\|x\| \geqq |x|$ for all x in A. Then B is a dual algebra and A and B have the same socle S (see Lemma 5.1 in [15]). Let $\{I_\lambda : \lambda \in \Lambda\}$ be the family of all minimal closed two-sided ideals of A. Then A is the topological direct sum of $\{I_\lambda : \lambda \in \Lambda\}$. For each $\lambda \in \Lambda$, let $B_\lambda = \mathrm{cl}(I_\lambda)$. Since B is a dual B^*-algebra, $B = (\sum_\lambda B_\lambda)_0$, the $B^*(\infty)$-sum of $\{B_\lambda : \lambda \in \Lambda\}$. Let L_λ be a minimal left ideal of I_λ. Since A and B have the same socle, L_λ is also a minimal left ideal of B_λ. As shown in the proof of Theorem 3.5, L_λ is a Hilbert space and B_λ can be considered as the algebra of all completely continuous linear operators on L_λ. Also $\|\cdot\|$ and $|\cdot|$ are equivalent on L_λ, i.e., there exists a constant $M_\lambda > 0$ such that

$$M_\lambda \|x\| \leqq |x| \leqq K\|x\| \qquad (x \in L_\lambda, \lambda \in \Lambda). \tag{4.2}$$

Put

$$M = \inf\{M_\lambda : \lambda \in \Lambda\}. \tag{4.3}$$

Of course, M may be zero. We give some examples for which N is not zero. Clearly if Λ is finite, then $M \neq 0$. Another example is the algebra $A = (\sum_\lambda \tau c(H_\lambda))_1$

given in [11, page 64]. By [9, page 261, Lemma (4.10.1)], there exists a hermitian minimal idempotent e_λ in A such that $L_\lambda = Ae_\lambda$. Then for all x in L_λ, $\|x\| = \|xe_\lambda\| \leqq |x|\,\|e_\lambda\|$. Since $\|\cdot\|$ is a cross norm on $\tau c(H_\lambda)$, $\|e_\lambda\| = |e_\lambda| = 1$ (see [9, page 289]). Therefore $\|x\| \leqq |x|$ for all x in $L_\lambda(\lambda \in \Lambda)$. Hence $M = 1$.

THEOREM 4.2. *Let A be a semi-simple annihilator Banach algebra with a complementor q which is a dense subalgebra of a B^*-algebra B. Suppose $\|\cdot\|$ majorizes $|\cdot|$ on A and the constant M in (4.3) is not zero. Then the mapping $p: R \to \mathrm{cl}((R \cap A)^q$ on the closed right ideals R of B is a complementor on B.*

PROOF. By [1, page 39, Lemma 3], $x \in \mathrm{cl}_A(xA)$ for all x in A. Let $\{f_\alpha : \alpha \in \nabla\}$ be a maximal orthogonal family of q-projections in A. In order that p be a complementor on B, it is sufficient to show that $y = \sum_\alpha f_\alpha y$ in $|\cdot|$ for all y in B by Lemma 2.2 and Theorem 3.2 in [16]. Since $M > 0$, by (4.2) we have

$$M\|x\| \leqq |x| \leqq L\,\|x\| \qquad (x \in L_\lambda, \lambda \in \Lambda). \tag{4.4}$$

Let ∇_0 be a finite subset of ∇ and write $\nabla_0 = \nabla_1 \cup \cdots \cup \nabla_n$, where $\{f_\alpha : \alpha \in \nabla_i\} \subset I_{\lambda_i}$ $(\lambda_i \in \nabla,\ i = 1, \cdots, n)$. Let $v \in B$ and $x \in L_{\lambda_i}$. Then $yx \in L_{\lambda_i} \subset I_{\lambda_i} \subset A$. Since q is a complementor on A, by Theorem 3.2 in [16], $a = \sum_\alpha f_\alpha a$ in $\|\cdot\|$ for all a in A and so by Corollary 3.5 in [16], there exists a constant D such that $\|\sum_{\alpha \in \Delta_i} f_i a\| \leqq D\|a\|$. Therefore by (4.4) we have

$$\Big|\sum_{\alpha \in \nabla_i} f_\alpha yx\Big| \leqq K\Big\|\sum_{\alpha \in \nabla_i} f_\alpha yx\Big\| \leqq KD\|yx\| \leqq KDM^{-1}|yx| \leqq N|yx|,$$

where $N = KDM^{-1}$. Since $\sum_{\alpha \in \nabla_i} f_\alpha y \in B_{\lambda_i}$, we have

$$\Big|\sum_{\alpha \in \nabla_i} f_\alpha y\Big| = \sup\Big\{\Big|\sum_{\alpha \in \nabla_i} f_\alpha yx\Big| : x \in L_{\lambda_i} \text{ and } |x| \leqq 1\Big\} \leqq N|y|.$$

Since B is a B^*-algebra, by [9, page 258, Lemma (4.9.21)]

$$\Big|\sum_{\alpha \in \nabla_i} f_\alpha y\Big| = \max\Big\{\Big|\sum_{\alpha \in \nabla_i} f_\alpha y\Big| : i = 1, \cdots, n\Big\} \leqq N|y|.$$

Since ∇_0 is an arbitrary finite subset of ∇, it follows from the proof of Theorem 5.2 in [15] that $\sum_\alpha f_\alpha y$ is summable in $|\cdot|$. Now by the proof of Theorem 3.2 in [16], $y = \sum_\alpha f_\alpha y$ in $|\cdot|$ and this completes the proof.

REMARK. Theorem 4.2 is a generalization of [11, page 63, Theorem 8.8]. We omit the proof of this implication, because this is not that important.

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