MOST EFFICIENT FLUCTUATION RESERVES

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Statutory Contingency Reserves and Surplus

In 1935 the New York Insurance Department introduced the concept of special contingency funds for certain types of insurance. Such requirements had first been introduced in 1925 for mutual workmen's compensation companies. Clear, consistent principles for these funds were not stated at the time, but their purpose seems to be to provide a cushion that may be used in time of serious financial difficulty.

In group life insurance, this fund is a "special contingency reserve" and is carried at the suggestion of the New York Department [1]). For mutual casualty, nonprofit hospitalization and medical indemnity plans, and for reciprocal insurers, the fund is treated as "special contingent surplus", and is a mandated substitute for the minimum capital required of stock insurers [2]). The U.S. federal Life Insurance Company Income Tax Act of 1959 recognizes special credits of a similar nature for health, non-participating life and group life and health insurance coverages.

In all of these cases, the accumulations or credits are defined by a designated percentage of premiums [3]). For the New York Department group life reserve and the Income Tax health and group life credit, a maximum is defined by a second percentage of the same annual premiums base. For the nonprofit plans' surplus, a similar

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[1]) Described in Eilers, Robert D. and Crowe, Robert M., *Group Insurance Handbook*, Richard D. Irwin, Inc., Homewood, Illinois, 1965, and also briefly discussed in the Society of Actuaries in "Unassigned Surplus and Contingency Reserves," *Transactions, Society of Actuaries*, Volume XIX, Chicago, Illinois, 1967.

[2]) A full discussion of these "surplus" items may be found in Haley, James B., "Special Contingent Surplus for Mutual and Nonprofit Companies," Chapter 10, *Examination of Insurance Companies*, Volume 4, New York Insurance Department, 1954.

[3]) For non-participating contracts, the percentage of premium credit for tax purposes may be replaced by a percentage of the increase in reserves, if greater

maximum in terms of premium is used. However, in the case of mutual casualty companies, and of reciprocal insurers, the "special contingency surplus" must be built up to a defined absolute amount equal to the amount that would be required as capital of a stock company; there are no statutory provisions for withdrawing any part of such a fund once it is accumulated.

Fluctuation Reserves

It is the thesis of this paper that the contingency funds referred to above, with the exception of those clearly contemplated as a replacement of capital stock, are existing examples of funds that should properly be held on many existing insurance lines. Since these funds will be viewed, in this paper, as reserves held only against the effects of random fluctuations in claim experience, they will be termed "fluctuation reserves". We shall offer two suggested principles for the design of such fluctuation reserves. It is also the contention of this paper that the rules that are now applied in the accumulation of contingency reserves and surplus are—at best—empirical settings within one of the many possible designs that such rules might follow.

Determination of optimal rules to be followed in the design of fluctuation reserves is not pursued here except in a single limited case. It is hoped that the statement of possible *desiderata* for such rules will stimulate further investigations of the broad class of possible fluctuation rules.

The purpose of a fluctuation reserve is, we believe, to smooth years of relatively large profit with years of relatively large loss; to stabilize the profits and losses that are charged to an insurer's surplus—or that are available for distribution as dividends or premium refunds. What is envisioned is the archetypal reservoir of water—that which keeps the excess above normal needs in times of plenty, and releases parts of such excess in times of need.

In this view, an improperly managed fluctuation reserve arises from either over- or under-funding. It may be characterized by long periods of inactivity—sitting either at capacity (and therefore not levelling profits) or at zero (and therefore not levelling losses). The precise nature of a well-balanced fluctuation reserve will depend on the annual claim distribution, margins for fluctuations

(in the premium actually charged), and in the legal restraints on the reserve actually held.

In a degenerate case, illustrating the latter consideration, if no limitation on the maximum of such reserve were imposed, and the insurer a) has recognized the long-term average cost of the coverage, and b) placed all realized profits into the reserve, eventually the reserve reaches a size from which, in a probability sense, absolute maximization of stability is reached—no profits are ever realized (all such being paid to the reserve) and no losses are realized (all being paid from the reserve when in excess of premiums).

From the preceding paragraph, the following principle is suggested for all fluctuation reserves:

a) The expected return to the insurer from a particular coverage be the same whether the fluctuation reserve is utilized or not.

This principle makes it impossible that the fluctuation reserve will absorb all of the profits of the insurance operation, and (likewise) insures that its function is strictly within the interests of both the insurer and the regulatory officials.

Limitations of the present study

In the present case, we shall study the following hypothetical distribution of *annual* claims:

Amount:	3	5	10	15	20	25	30
Probability:	0.2	0.3	0.2	0.14	0.08	0.04	0.04

It should be recognized that this distribution of annual claims is a dangerous one. While there is a substantial probability of claim years that are relatively "normal", there is a substantial probability that total annual claims will be well over twice the year's expected annual claims. This latter figure is equal to 10 units.

We shall also assume that the insurer has added a fluctuation margin of 30% to the net expected costs, and is charging a net premium of 13 units. Naturally, this additional margin will depend on the insurer's assessment of his available surplus, and be determined by a risk theoretic model for his probability of losing the available surplus.

Finally, we shall assume that the net premium of 13 units is net of profit, expenses, taxes, and other nuisance additions.

The following rules will be assumed for the operation of the fluctuation reserve:

1) All claims in excess of the year's premiums are first chargeable to the fluctuation reserve,

2) Additions to the reserve shall consist of a percentage of the "apparent profits" (premiums less not claims charged by item 1) or to free surplus),

3) The reserve operates without interest or marginal tax credits,

4) No minimum for the present reserve is established (although such a limitation might certainly be incorporated in an "optimal" set of reserve rules),

5) The maximum for the reserve is two times the expected annual claims (again, an optimal reserve structure might violate this rule), and

6) During years in which there is an "apparent loss", (claims in excess of premiums) such losses are first chargeable to the reserve and thereafter any excess is taken as a realized charge against surplus (= a "real loss").

Finally, it will be required that the reserve never assume non-integer values. If a reserve is theoretically required to hold an amount of "$n.d$", a reserve of "n" is to be established "1-d" of the time, and a reserve of "$n + 1$" is established "d" of the time. This latter consideration is a calculational nicety which does not change the thrust of the argument made.

It must be realized that the above rules for the formation of a fluctuation reserve are quite special, and that further investigation of fluctuation reserve rules should consider each of the present constraints free. In this study, the only free parameter is the specific percentage of "apparent profits" which should be added to the fluctuation reserve when such "profits" are available.

Computational

It is intuitively clear that there should be a certain probability that, after a long period with a level premium income and a constant expected claim distribution, the operation of a fluctuation reserve will have resulted in an amount of at least x being in it. Furthermore, it is intuitively obvious that some "states" of the reserve will be

more likely than others; if claims are relatively small (and full claims are chargeable to the reserve—and also fairly likely!), the reserve will likely not be at its maximum. On the other hand, if claims are rare, and fairly large, the reserve will frequently be in the process of building up to a given maximum, rather than at either full or empty levels. It is clear that, once "mature" conditions are reached, the operation of the reserve is affected only by the distribution of claims and the net premium (including fluctuation margins) charged—not the expected amount of profits from the line of business. For the current example, then, the operation of the fluctuation reserve should not change the 3 units of "risk profit" charged in excess of net expected claims. Principle a) should hold.

The methodology outlined above leads directly to a formulation of the illustrative problem given above as a Markov chain. We may easily study the situation where the claims expected in one year are independent of those in the preceding year and, for this case, the transition rules between states of the fluctuation reserve are easily displayed and computed. When a particular "rule" for the transition of a fluctuation reserve from one year to another is specified, and a certain "level" of expected claims is set, a "steady state" of the (eventual) reserve is created, and the current decision level for optimal use of the reserve may be determined.

In Table 1, we show the transition matrix that results from the operation of the fluctuation reserve rules in the preceding sections along with the accumulation rule in which *all* "apparent profits", full premiums less claims, are added to the reserve. Similar transition matrices may easily be created for other percentages of "apparent profits".

By studying the various operations of a "fluctuation reserve" of this nature, under various rules for the formation of reserve additions and charges, we can determine which is the "best" smoothing device of the realized profits. For, given the transition matrix, successively higher powers of it eventually result in the "steady state" in which the operation of the reserve is independent of the original level, and in which the principle expressed in a), the throwing off of realized profits in a steady way, is realized. This leads to our second, although tentative, principle:

TABLE 1

Transitional matrix from X to Y full premiums less claims

X↓	Y→0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	0 30			0 20					0 30		0 20										
1	0 30				0 20					0 30		0 20									
2	0 30					0 20					0 30		0 20								
3	0 16	0 14					0 20					0 30		0 20							
4	0 16		0 14					0 20					0 30		0 20						
5	0 16			0 14					0 20					0 30		0 20					
6	0 16				0 14					0 20					0 30		0 20				
7	0 16					0 14					0 20					0 30		0 20			
8	0 08	0 08					0 14					0 20					0 30		0 20		
9	0 08		0 08					0 14					0 20					0 30		0 20	
10	0 08			0 08					0 14					0 20					0 30		0 2
11	0 08				0 08					0 14					0 20					0 30	0 2
12	0 08					0 08					0 14					0 20					0 5
13	0 04	0 04					0 08					0 14					0 20				0 5
14	0 04		0 04					0 08					0 14					0 20			0 5
15	0 04			0 04					0 08					0 14					0 20		0 5
16	0 04				0 04					0 08					0 14					0 20	0 5
17	0 04					0 04					0 08					0 14					0 7
18		0 04					0 04					0 08					0 14				0 7
19			0 04					0 04					0 08					0 14			0 7
20				0 04					0 04					0 08					0 14		0 7

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b) A fluctuation reserve should minimize the standard deviation of the realized profits (after operation of the reserve) to the insurer.

Results

For the illustrative example outlined in the paragraphs above, calculations of the steady state of the reserve process were prepared for several percentages of "apparent profits" that might be required. The reserve rules were extended to include illustration of what happens when the maximum reserve is raised to twice annual net premiums (that is, expected claims plus the fluctuation margin) rather than twice expected claims. The net premium base is suggested by existing practice in the group life case.

The standard deviations of "realized profits" that would result from each of the rules tested are shown below in Table 2. For both maximums, the optimal percentage of "apparent profit" additions is about 60%, although the usual optimization "plateau" seems to apply in the 50%-75% range. For those interested in more detail, a sample of the computer output is available in the Appendix.

TABLE 2

Analysis of Illustrative Line of Insurance under various Fluctuation Reserve Rules

Percentage of Apparent Profits added to Reserve	Standard Deviation of Realized Profits with Maximum Fluctuation Reserve of	
	Twice Expected Claims	Twice Net Premiums
2%	7.145	7.145
25%	4.921	4.805
50%	4.081	3.774
60%	4.044	3.741
75%	4.094	3.822
100%	4.279	4.064

Summary

It is the present author's belief that the establishment of fluctuation reserves is justified by both solvency and adequacy needs of the state regulators. Yet they are used only in limited cases for solvency and for tax credit purposes. The demonstration given in the present paper, it is hoped, will stimulate others to examine specific lines of business to find the "maximumly efficient" rules for

Analysis of fluctuation reserve and realized profits with 60% of premiums less claims added to reserve

Steady state of reserve			*Realized profit/loss distribution*		
Reserve	*Prob. of state*	*Cumulative prob.*	*Profit*	*Prob. of profit*	*Cumulative prob.*
0	0.048298767441	0.048298767441	−17	0.001931950698	0.001931950698
1	0.006029102131	0.054327869572	−16	0.000241164085	0.002173114783
2	0.015406713089	0.069734582661	−15	0.000616268524	0.002789383306
3	0.007912545129	0.077647127790	−14	0.000316501805	0.003105885112
4	0.012073364627	0.089720492417	−13	0.000482934585	0.003588819697
5	0.021006595006	0.110727087423	−12	0.002772214498	0.006361034195
6	0.020467530244	0.131194617667	−11	0.001059865295	0.007420899490
7	0.021308824724	0.152503442392	−10	0.001468621513	0.008889521002
8	0.016234701669	0.168738144060	−9	0.000965889872	0.009855410874
9	0.029227152582	0.197965296642	−8	0.001652020688	0.011507431562
10	0.019484393685	0.217449690328	−7	0.005483540943	0.016990972505
11	0.021336386424	0.238786076752	−6	0.002154484837	0.019145457343
12	0.025922894463	0.264708971215	−5	0.003121805815	0.022267263157
13	0.020323122738	0.285032093953	−4	0.002095316587	0.024362579744
14	0.035092214888	0.320124308841	−3	0.003538643869	0.027901223613
15	0.022687188177	0.342811497018	−2	0.010129218317	0.038030441930
16	0.021943126694	0.364754623712	−1	0.004212657243	0.042243099172
17	0.028927667013	0.393682290725	0	0.257756900828	0.300000000000
18	0.021942351796	0.415624642522	1	0.101291743902	0.401291743902
19	0.050140359078	0.465765001600	2	0.030000637988	0.431292381890
20	0.023111448549	0.488876450149	3	0.191700600938	0.622992982828
21	0.023594311634	0.512470761784	4	0.136698754645	0.759691737473
22	0.027250729693	0.539721491477	5	0.011378751278	0.771070488751
23	0.022199629836	0.561921121314	6	0.026795829360	0.797866318111
24	0.071152278073	0.633073399386	7	0.011456478986	0.809322797097
25	0.023388510062	0.656461909449	8	0.117291882780	0.926614679877
26*	0.343538090551	1.000000000000	9	0.004677702012	0.931292381890
			10	0.068707618110	1.000000000000

* Twice annual premiums

S.D. of Profit = 3.741077140810

APPENDIX

Analysis of fluctuation reserve and realized profits with 75% of premiums less claims added to reserve

Steady state of reserve			*Realized profit/loss distribution*		
Reserve	*Prob. of state*	*Cumulative prob.*	*Profit*	*Prob. of profit*	*Cumulative prob.*
0	0.037008809223	0.037008809223	−17	0.001480352369	0.001480352369
1	0.003547528870	0.040556338093	−16	0.000141901155	0.001622253524
2	0.012171982876	0.052728320968	−15	0.000486879315	0.002109132839
3	0.006300406225	0.059028727193	−14	0.000252016249	0.002361149088
4	0.007165066263	0.066193793456	−13	0.000286602651	0.002647751738
5	0.007952528110	0.074146321566	−12	0.001798453493	0.004446205232
6	0.017561517886	0.091707839451	−11	0.000844361870	0.005290567102
7	0.017951515050	0.109659354501	−10	0.001204939917	0.006495507019
8	0.016158545719	0.125817900220	−9	0.000898358078	0.007393865097
9	0.028224061602	0.154041961822	−8	0.001415565115	0.008804302211
10	0.013561424984	0.167603386807	−7	0.003821262862	0.012630693073
11	0.013278968447	0.180882355254	−6	0.001517421763	0.014148114836
12	0.021943067764	0.202825423018	−5	0.002609541943	0.016717656778
13	0.016271971542	0.219097394560	−4	0.001801253188	0.018518909967
14	0.033696726661	0.252794121220	−3	0.003050036832	0.021568946798
15	0.021238197576	0.274032318797	−2	0.007209420442	0.028778367241
16	0.018467941511	0.292500260307	−1	0.003171451871	0.031949819112
17	0.026026714853	0.318526975160	0	0.291949437420	0.323899256531
18	0.018047736982	0.336574712142	1	0.086579659389	0.410478915921
19	0.050293955519	0.386868667661	2	0.161002211277	0.571481127198
20	0.021751014903	0.408619682564	3	0.135245346713	0.706726473910
21	0.022556118697	0.431175801261	4	0.012518767304	0.719245241215
22	0.027228547745	0.458404349006	5	0.010385458225	0.729630699440
23	0.019580781620	0.477985130626	6	0.027768544242	0.757399243682
24	0.074409448977	0.552394579603	7	0.011054409265	0.768452652947
25	0.023794176469	0.576188756073	8	0.142025262974	0.910478915921
26	0.423811243927	1.000000000000	9	0.004758835294	0.915237751215
			10	0.084762248785	1.000000000000

S.D. of Profit = 3.822471865388

Analysis of fluctuation reserve and realized profits with 100% of premiums less claims added to reserves

Steady state of reserve			*Realized profit/loss distribution*		
Reserve	*Prob of state*	*Cumulative prob*	*Profit*	*Prob of profit*	*Cumulative prob*
0	0 027003342095	0 027003342095	−17	0 001080133684	0 001080133684
1	0 003449777945	0 030453120041	−16	0 000137991118	0 001218124802
2	0 005607634404	0 036060754444	−15	0 000224305376	0 001442430178
3	0 008969634925	0 045030389370	−14	0 000358785397	0 001801215575
4	0 003111277906	0 048141667275	−13	0 000124451116	0 001925666691
5	0 006723865784	0 054865533059	−12	0 001349088315	0 003274755006
6	0 005834247814	0 060699780873	−11	0 000371361030	0 003646116037
7	0 011650202648	0 072349983522	−10	0 000690313482	0 004336429519
8	0 014362569221	0 086712552743	−9	0 000933288166	0 005269717684
9	0 025128745255	0 111841297998	−8	0 001129600926	0 006399318611
10	0 015645467984	0 127486765981	−7	0 003055040718	0 009454359329
11	0 009554900347	0 137041666328	−6	0 000891548162	0 010345907491
12	0 018290946477	0 155332612806	−5	0 001646256717	0 011992164209
13	0 011347788992	0 166680401798	−4	0 001745985123	0 013738149331
14	0 026930428292	0 193610830090	−3	0 002331269174	0 016069418506
15	0 015259644964	0 208870475054	−2	0 005554581674	0 021624000179
16	0 010875322102	0 219745797156	−1	0 001766917635	0 023390917815
17	0 029092534818	0 248838331975	0	0 478017422068	0 501408339883
18	0 014777798963	0 263616130937	1	0 037807296167	0 539215636049
19	0 052113468625	0 315729599563	2	0 012397846546	0 551613482595
20	0 019140051479	0 334869651042	3	0 115031752961	0 666645235556
21	0 010128067770	0 344997718812	4	0 013910510957	0 680555746513
22	0 033608335539	0 378606054350	5	0 006005309004	0 686561055517
23	0 013265651500	0 391871705850	6	0 031253790031	0 717814845548
24	0 081773743077	0 473645448927	7	0 008203537264	0 726018382812
25	0 018501356546	0 492146805473	8	0 168710706973	0 894729089785
26	0 507853194527	1 000000000000	9	0 003700271309	0 898429361095
			10	0 101570638905	1 000000000000

S D of Profit = 4 064240471850

fluctuation reserves within the confines of the present rules for "contingency reserves and surplus". All lines of business in a particular company are subject to fluctuation, the smoothing of individual results is of prime consideration to the actuary.

It is hoped that additional rules for fluctuation reserves will not be limited to the particular parameter studied in the present work — it is possible that entirely new classes of reserve formation rules will be established in the future, and that such rules will lead to even more efficient fluctuation reserves