Multiplication of Decimals.—The method of arranging the partial products illustrated in the accompanying example should offer no difficulty, if in explaining it we take the steps given below.

$$\begin{array}{r} 7{\cdot}63 \\ 31{\cdot}8 \\ \hline 228{\cdot}9 \\ 7{\cdot}63 \\ 6{\cdot}104 \\ \hline 242{\cdot}634 \end{array}$$

(i) *Multiplication by 10, 100, 1000, etc.*

It should be emphasised that in multiplying by 100, *e.g.* we do not move the decimal point, but move each digit of the number *two* places to the *left.*

(ii) *Division by 10, 100, etc.* To divide by 100 we move each digit two places to the *right,* keeping the position of the decimal point unchanged.

(iii) *Multiplication by an integer less than 10.* In multiplying 3·76 by 4 we say: "The 6 is 6 hundredths; multiplying by 4 we get 24 hundredths; we put the 4 below the 6 (in the hundredths place) and carry forward 20 hundredths as 2 tenths," etc.

$$\begin{array}{r} 3{\cdot}76 \\ 4 \\ \hline 15{\cdot}04 \end{array}$$

(iv) *Multiplication by 20, 30, 40, etc.* In multiplying 3·76 by 40 we multiply 3·76 by 4 and the product obtained by 10, *i.e.* we multiply by 4 and move each digit of the product as we obtain it *one* place to the left.

$$\begin{array}{r} 3{\cdot}76 \\ 40{\cdot} \\ \hline 150{\cdot}4 \end{array}$$

(v) *Multiplication by 200, 300, etc.*, is explained in a similar way.

(vi) *Multiplication by ·2, ·3, ·4, etc.* ·3 is $\frac{3}{10}$, so that we must multiply by 3 and divide by 10, *i.e.* we multiply by 3 and move each digit of the product as we obtain it *one* place to the *right.*

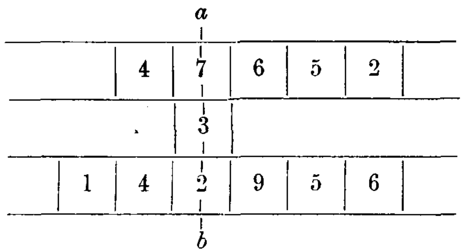
(vii) *Multiplication by ·02, ·03, etc.*, is explained in a similar way.

(viii) *The general case* may now be dealt with, and there will be no difficulty in placing the first digit of any partial product.

$$\begin{array}{r} 786{\cdot}423 \\ 4375{\cdot}268 \\ \hline 3145692{\cdot} \\ 6{\cdot}291384 \end{array}$$

(*a*) Let it be, for example, the partial product obtained when we multiply by the 4; this 4 is 4000; we therefore multiply by 4 and move each digit 3 places to the *left.* (*b*) Let the multiplier be the 8; this 8 is $\frac{8}{1000}$; we therefore multiply by 8 and move each digit 3 places to the *right.* It is obvious that because the 4 is 3 places to the *left* of the units digit we move each digit of the product obtained when we multiply by 4, 3 places to the *left*; and because the 8 is 3 places to the *right* of the units digit we move each digit of the product obtained when we multiply by 8, 3 places to the *right.*

The diagram illustrates a simple contrivance which would be helpful in exercising a class in the kind of work outlined above.



The digits are on square blocks of wood which can be moved to left or right in a slot, or withdrawn altogether, like the movable part of a slide rule. There are three slots parallel to one another. *ab* is a wire or bar fixed close to but above the blocks; the digit under it is the units digit. A pupil is asked to slip in and place properly in the uppermost slot the digits representing a given number 47·652; he is then asked to multiply (or divide) the number by 10, 100, ...; he does so by moving the series of blocks 1, 2, ... places to the left (or to the right). The block in the centre slot, carrying the 3 (the multiplier) is placed under the wire *ab* and blocks are slipped into the lowermost slot to represent the product. The 3 is then moved to a different position so as to represent 30, 300, ... ·3, ·03, ..., and the pupil is asked to correctly place the series of digits 142956, so that they will represent the product.

The placing of the decimal point to the *right* of the units digit is obviously unfortunate. We wish always to call attention to the position of a digit with reference to the *units digit;* yet the pupil is apt to think of the position of the digit with reference to the *decimal point.* For example, in the number 327·46 the 3 is two places to the left of the units digit and the 6 two places to the right; the pupil thinks of the 3 as three places to the left of the decimal point. Either of the notations $32\overset{\times}{7}46$, $32\dot{7}46$ would be, from this point of view, an improvement.

If the pupil is trained to think of the position of any digit of a number with reference to the *units digit*, he will have no difficulty in answering such questions as the following: "By what power of 10 must the number ·0638 be multiplied so as to bring the first significant figure into the units place?" "By what power of 10 must we divide 7382·7 so as to bring the first significant figure into the units place?"; or in understanding that "the characteristic of the logarithm of a number whose first significant figure is k places to the left (or right) of the units digit is k (or $-k$)." In establishing this rule, we simply assume that the logarithm of a number whose first significant figure is in the units place (and which therefore lies between 1 and 10) lies between 0 and 1.

Assume for example that

$$7{\cdot}382 = 10^{0{\cdot}8682};$$

then

$$738{\cdot}2 \quad = 10^{2{\cdot}8682},$$

because the 7 is *two* places to the left of the units digit, and in moving it (and the other digits) 2 places to the left, we have multiplied by 10^2, and

$${\cdot}07382 = 10^{-2+{\cdot}8682},$$

because the 7 is *two* places to the right of the units digit, and we have therefore divided 7·382 by 10^2.

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The Possible Error in a Quotient.—Let the quotient be $A \div B$ where the possible errors in A and B respectively are a and b.

The correct value of the quotient lies between $\dfrac{A+a}{B-b}$ and $\dfrac{A-a}{B+b}$.

Now $$\frac{A+a}{B-b} = \frac{A}{B}\left(1+\frac{a}{A}\right)\left(\frac{1}{1-\dfrac{b}{B}}\right)$$

$$= \frac{A}{B}\left(1+\frac{a}{A}\right)\left\{1+\frac{b}{B}+\frac{b^2}{B^2}+\frac{\dfrac{b^3}{B^3}}{1-\dfrac{b}{B}}\right\} \qquad (1)$$

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