CORRESPONDENCE.

THE VALUE OF A DEFERRED ANNUITY.

To the Editors of the Journal of the Institute of Actuaries.

SIRS,—In the excellent paper which he submitted to the Institute in November last with special reference to the valuation of mineral property, Mr. Carpmael states that " it is probably seldom in " actuarial practice that it is necessary to estimate the market value " of a deferred annuity certain", and that " in practice the valuation " of deferred annuities certain arises principally in connection with " mineral valuation." Actuaries have been concerned in the valuation of immediate and deferred annuities in connection with properties much more extensively than Mr. Carpmael assumes to be the case. The valuation of annuities immediate and reversionary arising from land and house property, and, it may be added, tithe rent-charge, is a matter of very great importance and one to which it may be hoped that the Institute will give its attention at some future time. It may be mentioned incidentally that surveyors have been divided in opinion as to the actuarial treatment of valuation problems with

regard to land and house property for the last 50 or 60 years at least and their differences have never been reconciled to this day.

It may be inferred from the discussion of Mr. Carpmael's paper that actuaries are generally in agreement with him in his criticisms of the Louis formulas as applied to the valuation of interests in property. The Louis (2) formula was presumably applied to the valuation of unleased minerals under the Finance Act 1909-10 and, moreover, it has been suggested that the use of the formulas should be extended to the valuation of interests in land and house property. If, therefore, a two-rate formula is unsuitable the Institute of Actuaries will render a great service to the community if its members expose the fallacies involved in its application to such purposes.

The advocates of the two-rate formulas assume that in the long run investors in mines obtain no greater return than from gilt-edged securities, but if so investors of this class are singularly unfortunate. It should be remembered that gilt-edged securities give a low yield because so frequently they alone are available to Trustees, but every financier of experience knows that a higher yield can be obtained with reasonable safety from securities which do not come within the confines of the Trustee Acts, as indeed the records of well-managed Insurance and Financial Trust Companies conclusively prove.

It seems to have been assumed that mineral surveyors are overwhelmingly in favour of the two-rate formulas, and in fact Prof. Louis goes so far as to assert that a valuation of interests in minerals by a one-rate formula " would be rejected immediately by " every mineral valuer of experience." The assumption is very questionable and in any case mineral surveyors of the widest experience have been bitterly opposed to the use of the two-rate formulas.

Some years ago one of the most experienced mineral surveyors in this country and a member of an eminent firm, was called upon to value a deferred interest in minerals for the purposes of estate duty. No one knew better than he the real market value of the interest and he was full of indignation when confronted by the Official Valuer, armed with the two-rate formula which produced a valuation greatly exceeding his own. In the course of the discussion he put the following case to the valuer : A has an immediate interest for t years and B for the succeeding n years. The official two-rate formulas for immediate and deferred annuities valued independently were respectively $\dfrac{s_{\overline{t}|}^{(i)}}{1 + js_{\overline{t}|}^{(i)}}$ and $\dfrac{s_{\overline{n}|}^{(i)}}{1 + js_{\overline{t+n}|}^{(i)}}$ where $i =$ the reproductive rate and $j =$ the remunerative rate of interest, but the value of the two interests over the $t + n$ years thus obtained was substantially greater than the value of the undivided interest for the same term, namely, $\dfrac{s_{\overline{t+n}|}^{(i)}}{1 + js_{\overline{t+n}|}^{(i)}}$.

Thus a dilemma arose, for the value of the two interests to a purchaser could not be greater than the value of the undivided interest. The official valuer caught out by this inconsistency instead of retiring from the field remained to argue that as certain

assumptions had to be made as to a sinking fund the value of A's

interest must be reduced to $\dfrac{s_{\overline{t+n}|}^{(i)} - s_{\overline{n}|}^{(i)}}{1 + js_{\overline{t+n}|}^{(i)}}$. In this way the value of

the two interests taken together was made to agree with the value

of the whole *i.e.,* $\dfrac{s_{\overline{t+n}|}^{(i)} - s_{\overline{n}|}^{(i)}}{1 + js_{\overline{t+n}|}^{(i)}} + \dfrac{s_{\overline{n}|}^{(i)}}{1 + js_{\overline{t+n}|}^{(i)}} = \dfrac{s_{\overline{t+n}|}^{(i)}}{1 + js_{\overline{t+n}|}^{(i)}}$ but the

valuer was thus driven from one absurdity to another, for obviously
A would not be disposed to accept the reduced value simply because
B possessed an interest for the succeeding term of n years. A could
not in fact be expected to concern himself with eccentric theories as
to the necessity of a fictitious sinking fund whether such views were
held by the purchaser or anyone else. The two rates employed in
this case were 8 and 3 per-cent, and if $t = 1$ and $n = 19$ the resulting

values as obtained by the formulas $\dfrac{s_{\overline{t+n}|}^{(i)} - s_{\overline{n}|}^{(i)}}{1 + js_{\overline{t+n}|}^{(i)}}$ and $\dfrac{s_{\overline{n}|}^{(i)}}{1 + js_{\overline{t+n}|}^{(i)}}$ were

such that A's interest was discounted at 80 per-cent and B's interest at
9·4 per-cent. If $t = 2$ and $n = 18$ the rates were 51 and 8·9 respectively.

On the official side much stress was laid upon the fact that the
two-rate formulas were to be found in Mr. George King's " Theory
of Finance", but on being appealed to Mr. King promptly replied that
these formulas are quite unsuitable for the valuation of properties
and he was whole-heartedly in agreement with the views to which
general expression was given by actuaries in the discussion of
Mr. Carpmael's paper, namely, that the valuation of interests in
minerals should be made by suitable one-rate formulas. On the
27 April 1911 he made a report on a mineral case and summed up his
argument as follows :

" I think that the only proper way to deal with these matters is
" to fix on a rate of interest that is suitable for the particular
" transaction in hand, and to keep it homogeneous throughout the
" whole of the calculations. Thus, if the market value of the
" property is £853 as given by the 8 per-cent and 3 per-cent rates of
" interest for 20 years, then we might fix on 10 per-cent to be the
" true rate of interest, and work out everything homogeneously at
" that rate. That gives a value of £851, which is very nearly the
" £853 brought out by the special formula. Here we see that the
" borrower would think that he was only paying 8 per-cent, but that
" in reality the interest to him is 10 per-cent.

" When the transactions are carried out thus homogeneously
" there is no need to set up a sinking fund. A schedule would be
" prepared at the outset similar to that given on page 33 of the third
" edition of my Theory of Finance. The table includes the interest
" contained in each payment of the annuity, and the principal also
" contained therein, and each year the investor would write off the
" principal of that year from his security, so that his books would
" show a diminishing investment as in column 5 of the table."

Yours faithfully,

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