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Corrigendum: '*An embedding theorem for abelian monoidal categories*, Compositio Math. 132 (2002), 27–48'

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The discussion at the beginning of § 3.5 is incorrect. Since A^{op} is not an ind-category but rather a pro-category, the tensor product on it preserves limits but not colimits; hence one cannot extend the monoidal structure from A^{op} to ${}_R\mathsf{Mod}$ as indicated there. Therefore, Theorem 3.2 is merely a conjecture. The following weaker form of Theorem 3.2 holds true.

Theorem A. *Let C be a small Abelian monoidal category with the tensor product being exact. Assume that C has an injective finite cogenerator J (i.e. each object of C is a subobject of a finite direct sum of copies of J). Then C^{op} admits a right exact monoidal embedding into the category ${}_R\mathsf{Mod}_R$, $X \mapsto \mathrm{Hom}_{\mathsf{C}}(X \odot J, J)$, where $R := \mathrm{Hom}_{\mathsf{C}}(J, J)$.*

Proof. The functor $\mathrm{Hom}_{\mathsf{C}}(-, J) : C^{\mathrm{op}} \to {}_R\mathsf{Mod}$ defines an equivalence between C^{op} and the category of finitely generated left R-modules. Consequently, it extends to an equivalence between the ind-category of C^{op} and ${}_R\mathsf{Mod}$. Thus we can extend the tensor product on C^{op}, which is exact, to an exact tensor product on ${}_R\mathsf{Mod}$. Hence we can apply Theorem 2.4 and obtain a monoidal right exact embedding $\omega := \mathrm{Hom}_{\mathsf{C}}(- \odot J, J) : \mathsf{C}^{\mathrm{op}} \to {}_R\mathsf{Mod}_R$. □

Consequently, Theorem 3.3, the proof of which relies on Theorem 3.2, is merely a conjecture. The following weaker form of it is a consequence of Theorem A above.

Theorem B. *Let C be a small Abelian monoidal rigid category. Assume that C has an injective finite cogenerator. Then C admits an exact monoidal embedding into the category of bimodules over a ring. Further, the embedding is extendable to an exact embedding of the ind-category of C, which commutes with colimits.*

Proof. The proof is the same as that of Theorem 3.3. □

Examples of categories satisfying the conditions of Theorem B are finite tensor categories, studied in [EO04].

The results of § 4, which rely on Theorem B, therefore hold true only under the assumption that C has finitely many simple objects. In fact, the tensor category constructed in [Del90, 2.19] is a counterexample to the statement of Proposition 4.1 and of Theorem 4.4.

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