SANDRA MÜLLER (*formerly* UHLENBROCK), ***Pure and Hybrid Mice with Finitely Many Woodin Cardinals from Levels of Determinacy***, Westfälische Wilhelms-Universität Münster, Germany, 2016. Supervised by Ralf-Dieter Schindler. MSC: primary 03E45, secondary 03E60, 03E55. Keywords: inner models, Woodin cardinals, projective determinacy, hybrid mice.

Abstract

Mice are sufficiently iterable canonical models of set theory. Martin and Steel showed in the 1980s that for every natural number n the existence of n Woodin cardinals with a measurable cardinal above them all implies that boldface $\boldsymbol{\Pi}^1_{n+1}$ determinacy holds, where $\boldsymbol{\Pi}^1_{n+1}$ is a pointclass in the projective hierarchy. Woodin and Neeman (in the late 80s and early 90s) then proved an exact correspondence between mice and projective determinacy. Woodin showed that boldface $\boldsymbol{\Pi}^1_{n+1}$ determinacy implies that the mouse $M_n^\#(x)$ with n Woodin cardinals exists and is ω_1-iterable for all reals x, the converse is due to Neeman. In the first part of this thesis we prove this implication of the result, which is a so far unpublished result by W. Hugh Woodin. In fact, the following theorem is shown in the thesis.

THEOREM 1. *Let $n \geq 1$ and assume there is no $\boldsymbol{\Sigma}^1_{n+2}$-definable ω_1-sequence of pairwise distinct reals. Moreover, assume that $\boldsymbol{\Pi}^1_n$ determinacy and Π^1_{n+1} determinacy hold. Then $M_n^\#$ exists and is ω_1-iterable.*

As a consequence, we can obtain the following Determinacy Transfer Theorem for all levels n.

THEOREM 2 (Determinacy Transfer Theorem). *For $n \geq 1$, $\boldsymbol{\Pi}^1_{n+1}$ determinacy is equivalent to $\Game^{(n)}(< \omega^2 - \boldsymbol{\Pi}^1_1)$ determinacy.*

Following this, we consider pointclasses in the $L(\mathbb{R})$-hierarchy and show that determinacy for them implies the existence and ω_1-iterability of certain hybrid mice with finitely many Woodin cardinals, which we call $M_k^{\Sigma,\#}$. These hybrid mice are like ordinary mice, but equipped with an iteration strategy for a mouse they are containing, and they naturally appear in the core model induction technique. The results we proved are in fact more general because they hold for an arbitrary adequate pointclass Γ which is $\mathbb{R}$-parametrized and has the scale property and a premouse $\mathcal{N}$ capturing certain sets of reals which has an iteration strategy $\Sigma \in \forall^{\mathbb{N}}\Gamma$ which condenses well. We apply these results to the following setting in the $L(\mathbb{R})$-hierarchy.

THEOREM 3. *Let $\alpha < \beta$ be ordinals such that $[\alpha, \beta]$ is a weak Σ_1-gap, let $k \geq 0$, and let $A \in \Gamma = \boldsymbol{\Sigma}_n(J_\beta(\mathbb{R})) \cap \mathcal{P}(\mathbb{R})$, where $n < \omega$ is the least natural number such that $\rho_n(J_\beta(\mathbb{R})) = \mathbb{R}$. Moreover, assume that every $\boldsymbol{\Pi}^1_{2k+5}\Gamma$-definable set of reals is determined. Then there exists an ω_1-iterable hybrid Σ-premouse $\mathcal{N}$ which captures every set of reals in the pointclass $\Sigma^1_k(A)$ or $\Pi^1_k(A)$.*

Abstract prepared by Sandra Müller (formerly Uhlenbrock)
E-mail: mueller.sandra@univie.ac.at
URL: http://www.logic.univie.ac.at/~smueller/UhlenbrockThesis.pdf

WILLIAM CHEN, ***Some Results on Tight Stationarity***, University of California, Los Angeles, USA, 2016. Supervised by Itay Neeman. MSC: 03E05. Keywords: mutual stationarity, pcf theory, Prikry forcing, singular cardinals, tight stationarity, tree-like scales.

Abstract

Fix an increasing sequence of regular cardinals $\langle \kappa_n : n < \omega \rangle$. Mutual and tight stationarity are properties akin to the usual notion of stationarity, but defined for sequences $\langle S_n : n < \omega \rangle$ with $S_n \subseteq \kappa_n$. This work focuses particularly on tight stationarity, providing a new characterization for it and comparing it to other concepts of stationarity.

Starting from a pcf-theoretic scale, we define a transfer function mapping a sequence of subsets to a single subset of a certain regular cardinal, the length of the scale. The transfer

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function preserves stationarity, in the sense that a sequence is tightly stationary if and only if it is mapped to a stationary subset.

Using this characterization, we explore the question of whether it is consistent that there exists a sequence of cardinals for which every stationary sequence (i.e., a sequence of subsets, each of which is stationary in the corresponding cardinal) is tightly stationary, and prove some results which give a negative answer in certain cases. We prove that adding Cohen reals introduces stationary sequences which are not tightly stationary, and in the extension by adding uncountably many Cohen reals, every sequence of cardinals has a stationary but not tightly stationary sequence. From a tree-like scale we construct a sequence of stationary sets that is not tightly stationary in a strong way, namely, its image under the transfer function is empty.

Investigating this question in the Prikry model, we define the notion of a forgetful sequence and prove that every forgetful sequence of cardinals has a stationary, not tightly stationary sequence. Along the way, we will analyze the scales which appear in the Prikry model.

Then we consider the question of Cummings, Foreman, and Magidor of whether it is consistent that there is a sequence of cardinals on which every mutually stationary sequence is tightly stationary. We prove that it is consistent that there is no such sequence of cardinals. This uses a supercompact version of a construction adapted from Koepke which ensures that every stationary sequence is mutually stationary, provided that there is enough space between successive cardinals of the underlying sequence. Furthermore, this property of the model is indestructible under further Prikry forcing, which suggests that it is difficult to obtain a positive answer to the CFM question. The results in this section were obtained jointly with Itay Neeman.

Finally, we explore the combinatorics of tight stationarity. This leads to the notion of a careful set, which is a strengthening of being in the range of the transfer function. We produce a model where there is a singular cardinal for which all subsets of the successor are careful, which suffices to prove a splitting result for tightly stationary sequences. Using a version of the diagonal supercompact Prikry forcing, we obtain such a model where the singular cardinal is strong limit. These results start from a model with a continuous tree-like scale on the singular cardinal.

Abstract prepared by William Chen
E-mail: chenwb@gmail.com
URL: https://escholarship.org/uc/item/6c7392f3

Zanyar Anwer Ameen, ***Finitely Additive Measures on Topological Spaces and Boolean Algebras***, University of East Anglia, UK, 2015. Supervised by Mirna Džamonja. MSC: Primary 28A05, Secondary 28A60, 28A12 and 28A75. Keywords: uniformly regular measure, separable measure, Jordan algebra, Jordan field, charge algebra and Maharam Theorem.

Abstract

The thesis studies some problems in measure theory. In particular, a possible generalization corresponding to Maharam Theorem for finitely additive measures (charges). In Chapter one, we give some definitions and results on different areas of Mathematics that will be used during this work.

In Chapter two, we recall the definitions of nonatomic, continuous and Darboux charges, and show their relations to each other. The relation between charges on Boolean algebras and the induced measures on their Stone spaces is mentioned in this chapter. We also show that for any charge algebra, there exists a compact zero-dimensional space such that its charge algebra is isomorphic to the given charge algebra.

In Chapter three, we give the definition of Jordan measure and some of its outcomes. We define another measure on an algebra of subsets of some sets called Jordanian measure, and investigate it. Then we define the Jordan algebras and Jordanian algebras and study some of their properties.

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