INTERNAL STRUCTURE AND ROTATION: SEISMIC INVERSIONS

THEORETICAL SOLAR MODELS

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Abstract. In the last two decades the large amount of accurate helioseismic data has open a new possibility to sound the stellar interiors and to test the stellar evolution theory. This has particularly impulsed a critical reexamination of the basic assumptions and physical inputs in solar modelling. The present status of theoretical solar models is presented and discussed in relation with the observational constraints: helioseismology, lithium depletion and measured neutrino fluxes.

1. Introduction

The Sun is a normal star of one solar mass which is burning its central hydrogen. So theoretical solar models are obtained using the theory of stellar evolution which attempts to describe the evolution of the structure of a star during its life. A success of this theory was to explain the statistical properties of stars like the accumulation of stars along the so called main sequence in the Herzsprung-Russell diagram relating the stellar luminosity and effective temperature. But up to recently this theory remained not much constrained. In the last 30 years two possibilities to test the solar interior appeared: solar neutrino measurements and helioseismic observations.

The results of neutrino experiments have much questioned the validity of solar models hence that of the stellar evolution theory. This theory relies on simplifying basic assumptions. The star, hence the Sun, is assumed spherically symmetric: rotation, magnetic field, macroscopic motions are ignored. The Sun is in hydrostatic equilibrium, i.e. pressure balances gravity. It is in thermal equilibrium: the energy loss at the surface by radiation is equal to the energy produced in the core. The energy source is nuclear reactions, mainly the hydrogen burning in the solar case. The tempera-

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ture gradient along the radius results from the energy transport, assumed to be stationary, from the center to the surface. This transport occurs by means of radiation except in the convectively unstable zone, which for the Sun extends on the outer 30% of the radius . The radiative transport is controlled by the atomic absorption coefficients which together with the chemical composition determine the opacity of the stellar material. In convection zone the energy transport is modeled by the so called mixing length theory of the convection. It introduces a free parameter, α that measures the convection efficiency.

The solar evolution results from changes of chemical composition, mainly due to the fusion of hydrogen into helium. The composition changes also in the radiative zones of the Sun, due to the element diffusion which is driven by the gradients of the structure. The evolution is started from a chemically homogeneous model either during the pre main sequence contraction phase or on the zero age main sequence. The initial abundances are assumed those observed at the photosphere. Mixing processes outside the convectively unstable zone, which is assumed perfectly mixed, are generally ignored. In the case of the Sun the knowledge of the luminosity, radius and age allows to obtain with the above assumptions a theoretical solar model.

The large amount of very accurate helioseismic data has open a new possibility to sound the solar and stellar interiors. This has impulsed a critical reexamination of the basic assumptions and of the physical inputs in solar and stellar modelling. A lot of work has been made specially on the sensitivity to the physics of the solar structure, oscillations and neutrino fluxes. This makes impossible to give an exhaustive presentation. Many complete reviews on the stellar and solar modelling have been presented recently (Turck-Chièze *et al.* 1993, Berthomieu 1996, Christensen-Dalsgaard *et al.* 1996, Lebreton & Baglin 1996, Roxburgh 1996; see also the references therein). This review is devoted to theoretical solar modelling and direct comparisons with helioseismic data, while the inferred seismic model is discussed in these proceedings by Basu (see also e.g. Gough *et al.*, 1996)

The recent development concerning the basic input physics (Section 2) as well as the present status of theoretical solar models (Section 3) are presented. The comparison between the theoretical oscillation frequencies and the helioseismic data is given in Section 4. Finally the solar neutrino problem is briefly considered (Section 5).

2. Physics of Solar Models

Given the basic assumptions of the stellar evolution theory, the structure of the model is determined by the microscopic properties of the solar material: the equation of state (EOS) which relates the pressure and thermodynamical quantities to the density, temperature and chemical composition, the opacity, the nuclear reactions rates and microscopic diffusion parameters. There have been important improvements in the description of the microscopic physics of the solar interior. On the contrary the macro-physics, needed to describe motions like convection, remains poorly accounted.

Opacity

The opacity of the stellar interior depends both on the radiative atomic parameters and on the composition of the stellar mixture. Significant revisions of the opacities calculations have been obtained by two groups, with results in good agreement. The OPAL project (e.g. Rogers 1994) and the OP project (e.g. Seaton *et al.* 1994) have obtained an increase of opacity by over a factor 3 compared to earlier calculations for temperature from 100 000 K to 10^6K due to improved atomic physics for partially ionized Iron. Additional improvements in the bound bound transition of Iron using intermediate coupling rather than pure LS coupling led to further increase that results in successful modelling of Cepheid stars.

This increase of opacity has much changed the solar structure and oscillation frequencies providing deeper convection zone and theoretical frequencies closer to the observed ones (e.g. Guzik & Cox 1991, Bahcall & Pinsonneault 1992, Guenther *et al.* 1992, Berthomieu *et al.* 1993). It results also in a strong increase of the initial helium abundance estimated from the solar calibration from 0.24 to 0.28. It was suspected by Christensen-Dalsgaard *et al.* (1985) and Korzennik & Ulrich (1989) that such an opacity increase will improve the agreement with the observations. Last updated OPAL opacities with improvements in physics, specially the equation of state, and refinements of the element abundances, including more metals, have shown that the decrease due to a better EOS is almost compensated by the increase due to the new composition except near the base of the convection zone (log T=6.2) (Iglesias & Rogers 1996).

Low temperature opacities, less accurate due to molecular contributions, have been recomputed by several groups (Kurucz 1991, Alexander Ferguson 1994, Neuforge 1993, Sharp & Turck-Chièze 1996). They influence much the structure of the superficial layers, hence the oscillation frequencies, but they have no significant effect on the depth of convection zone as long as the model is calibrated to the solar radius.

Equation of state

The solar plasma at first approximation is a mixture of almost fully ionized perfect gases except close to the surface around T$\sim$ 10^5K. There is very small departure from perfect gas due to dynamical interaction between the components of the plasma. But the very accurate helioseismic data allow to study these small effects due to the sensitivity of the oscillations specially to the Γ_1 index which relates the changes in pressure and density of the oscillations. Γ_1 depends much on chemical composition in the partially ionized zones, thus it provides a possibility of diagnostic of the helium content in the convection zone (Gough 1984).

Two detailed description of the EOS MHD (Mihalas *et al.* 1990) and OPAL (Rogers *et al.* 1996) have been developed in parallel with the OP and Livermore opacity calculations. There is a good agreement between the two EOS, except a pressure difference of 1.2 percents around 100 000K (Rogers *et al.* 1996). Its origin has been explained by Baturin *et al.* (1996), as the manifestation of some "τ" correction for the Debye-Hückel theory which is used in the MHD formalism. The fact that such a small difference can be seen from helioseismology (as shown by Guzik *et al.* 1996) open to plasma physicists a possibility to test their theory in a domain not accessible to laboratory experiments.

Microscopic diffusion

In the radiative zones of the Sun, in absence of macroscopic motions, the composition changes due to the element diffusion which is mainly driven by pressure gradient, temperature gradient and composition gradients. The effect of radiative pressure is negligible in the solar case (e.g. Vauclair 1996). The heavier elements tend to sink towards the center and sharp composition gradients are smoothed by the diffusion. In convection zones the motion is assumed rapid enough to make the chemical composition homogeneous and to inhibit the segregation of elements through diffusion process.

Computation of diffusion velocities are mostly based on the work of Burgers (1969) who provided a complete set of equations to describe the evolution of a multicomponent fluid. This diffusion process has been introduced in solar modelling first by Cox *et al.* (1989). Different formulations have been used: Thoul *et al.* 1994 solve exactly the Burgers equations and then represent the results by simple analytical functions, while Michaud & Proffitt (1993) search analytical solutions by approximations. Their descriptions agree within 15% according to Bahcall & Pinsonneault (1995).

Uncertainties of the solar macro-physics

Let us now point out the main uncertainties of the solar structure which results from the rough description of the solar macro-physics.

First, at the surface, the structure of the super adiabatic layer where almost the whole energy is transported by convection depends much of the theory of convection which gives the temperature gradient. The convection flux is estimated from mixing length theory which has no real physical basis. One ignores the turbulent pressure which represents 10% of the total pressure and which lowers the gas pressure and density (e.g. Balmforth 1992, Kosovichev 1995). A first step towards a more realistic description of the convection has been made using Canuto & Mazzitelli (1991) theory of convection (see Basu & Antia 1994) to compute solar models (Paterno *et* *al.* 1993). This theory tries to account for the whole spectrum of turbulent eddies. Next improvement may come from time dependent hydrodynamical simulations (e.g. Brummell *et al.* 1995) which may lead to improved parameterized formulations of the convection and to better description of the convection in solar conditions.

Concerning the radiative interior, it is likely that convective motions penetrate below the convection zone changing the temperature gradient (Zahn 1991). Different types of mixing can also take place in the radiative interior. Induced gravity waves may lead to significant mixing (Montalban 1994). Additional mixing may be caused by rotationally induced instabilities related to meridional circulation or to the spin down of solar interior from initial rapid rotation (Zahn 1997). These mixing processes can counteract the diffusion processes and result in modification of the abundance profile. Possible mixing of the nuclear core will change the evolution by bringing more hydrogen in the solar core and will result in a lower central temperature as proposed by Schatzman in relation with the solar neutrino problem (see Section 5).

3. Properties of Theoretical Solar Models

The computed model has to satisfy global solar constraints. The mass, radius and luminosity of the Sun are known with a substantial accuracy. The solar age has been estimated from radioactive datation of meteorites and spectroscopic measurements provide detailed chemical abundances of the solar photosphere relatively to the hydrogen. This is usually measured by the ratio of the mass fraction of heavy elements Z to the mass fraction in hydrogen X. The helium abundance Y is unknown.

Hence the solar model is calibrated to solar age, radius and luminosity by adjusting the two unknown parameters, Y initial helium abundance and the convection parameter α. Finally the calibrated model is determined by the input physics and the input parameters.

TABLE 1. Global solar constraints

$M_\odot$	(1.9891± 0.0004)	10^{33} g	planetary motions
$R_\odot$	(6.9599±0.001)	10^{10} cm	measure of visible surface
$L_\odot$	(3.846±0.005)	10^{33} erg s−1	Willson *et al.* (1988)
$t_\odot$	(4.52±0.004)	10^{9} y	Wasserburg (1995)
$(Z/X)_\odot$	0.0245±0.0015		Grevesse & Noels (1993)

The properties of some solar models computed with updated physics and with and without microscopic diffusion by P. Morel with CESAM code

TABLE 2. Global characteristics of theoretical solar models (from Morel *et al.* 1997a) compared to observed values: the helium content of the outer layers Y_{surf} and the depth of the convection zone r_{ZC} inferred from helioseismic data (see respectively Basu & Antia 1995 and Christensen-Dalsgaard *et al.* 1991); the lithium photospheric abundance at solar age $Li_{\odot}$ (=log (n_{Li}/n_H) with $n_H=10^{12}$); the measured neutrino capture rates Φ_{Ga} (Hampel *et al.*, 1996), Φ_{Cl} (Davis, 1993) and Φ_{Ka} (Fukuda *et al.*, 1996) (see Section 5); the low degree frequencies differences $\overline{\delta\nu_{02}}$ and $\overline{\delta\nu_{13}}$ characterizing the solar core properties correspond to a mean of recent observations (see Section 4).

	S	D	$D_{massloss}$	$\odot$
Y_{init}	0.268	0.274	0.273	–
Y_{surf}	0.268	0.245	0.0245	0.246; 0.249
$r_{ZC}/R_{\odot}$	0.723	0.711	0.711	0.713±0.003
T_c (10^6K)	15.38	15.65	15.62	–
ρ_c (g cm^{-3})	146.5	151.2	150.6	–
Y_c	0.617	0.638	0.639	–
Li_{ZAMS}	3.08	2.95	3.13	–
$Li_{\odot}$	3.08	2.88	1.92	1.12
Φ_{Ga} (SNU)	121± 9	130	144	$69^{+7.8}_{+8.1}$
Φ_{Cl} (SNU)	6.2± 2	8.27	8.93	2.55±0.23
Φ_{Ka} (10^6cm^{-2} s^{-1})	4.49± 1.4	6.20	6.67	2.8±0.4
$\overline{\delta\nu_{02}}$($\mu$Hz)	9.34	9.09	9.13	9.01±0.05
$\overline{\delta\nu_{13}}$($\mu$Hz)	16.40	16.04	15.98	15.90±0.08
P_0(mn)	36.45	35.75	35.76	–

are given in Table 2. These models are computed with OPAL equation of state and opacities, nuclear reaction rates from Caughlan & Fowler (1988) (see Morel *et al.* 1997a for details). The initial helium abundance Y_{init} required to calibrate the Sun at the present luminosity and radius is slightly larger for models with microscopic diffusion. The main effect of settling is a decrease of the surface helium abundance Y_{surf} by about 10%, in agreement with helioseismic measurement $Y_{\odot} \sim 0.25$ (see Basu these proceedings); there is an increase of the depth of the convection zone to a value close to the depth estimated from the helioseismology $r_{ZC}/R_{\odot} \sim 0.713$ (see Basu these proceedings). As the helium sinks toward the center, Y_c and ρ_c increase, hence X_c decreases which requires a higher central temperature to obtain the solar luminosity at solar age. Consequently the predicted neutrino capture rates are larger when helium settling takes place and this process slightly increases the discrepancy with the neutrino experimental measurements (see Section 4).

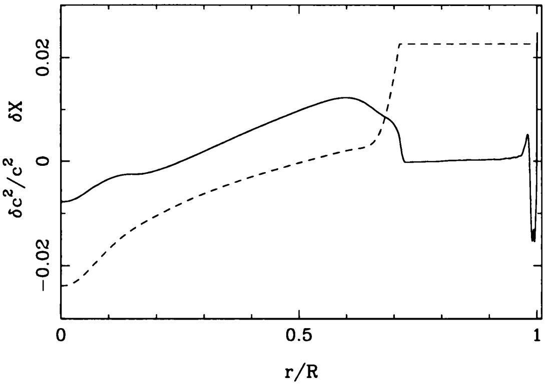

Figure 1. Difference of hydrogen content (dashed line) and relative square sound speed difference (full line) between the models with and without microscopic diffusion D and S, as a function of the solar radius.

Figure 1 illustrates the effect of the microscopic diffusion by the variation along the solar radius of the difference in hydrogen content and the relative sound speed difference between solar models with and without diffusion. Dominant effect is settling of Helium out of the convection zone. It results an increase in hydrogen content in the convection zone leading to an increase of its depth. The square of sound speed difference presents a large bump of about 1% in the layers below the convection zone.

Now many groups have computed models including settling of Helium (Proffitt & Michaud 1991, Bahcall & Pinsonneault 1992, Christensen-Dalsgaard *et al.* 1993, Chaboyer *et al.* 1995a, Gabriel & Carlier 1997) or settling of Helium and of heavier elements (Cox *et al.* 1989, Proffitt 1994, Bahcall & Pinsonneault 1995, Henney & Ulrich 1995, Basu *et al.* 1996, Morel *et al.* 1996 - 1997, Richard *et al.* 1996). The comparison of these models reveals a rather good agreement even if different formulations of the diffusion have been used.

Of course uncertainties still remain on the diffusion coefficients whose consequences on solar modelling have to be discussed (see Gabriel & Carlier in the posters proceedings of this meeting). Nevertheless microscopic diffusion can be now considered as standard assumption in solar modelling. Another uncertainty of all these models results from the fact that the available opacity tables are computed for fixed relative chemical compositions of the heavy elements entering in the considered mixture, while these values vary in time and space due to the nuclear burning and to the different velocity of diffusion of each element. A detailed discussion of that is given in Morel *et al.* (1997a).

These solar models computed according the standard assumptions do not account for the observed lithium depletion in the solar photosphere at the solar age: as shown in Table 2, the theoretical lithium abundance at solar age for the models with or without diffusion is much larger than the observed one by more than two orders of magnitude. The lithium can be nuclearly destroyed at a temperature $T=10^6$K higher than the temperature at the base of the convection zone of actual solar models. Thus its destruction requires to bring the lithium towards hot regions, either by an extension of the convection zone by overshoot or by some mixing or transport processes beneath the CZ at some phase of solar evolution. In solar models including penetrative convection, some lithium depletion occurs, slightly enhanced by diffusion, but to much smaller value than the observed one, unless significant penetrative convection is introduced (Ahrens *et al.* 1992, Morel *et al.* 1996).

Rotationally induced mixing has been introduced in solar modelling particularly in order to try to explain the observed lithium depletion. The combined effects of microscopic diffusion and rotational mixing treated through some effective diffusion coefficients have been investigated (Chaboyer *et al.* 1995b, Richard *et al.* 1996). The mild mixing occurring below the convection zone inhibits the helium diffusion in the outer layers resulting in a slightly thinner convection zone. The observed lithium abundance at the solar age is reproduced by adjustment of some parameters which characterize the amount of mixing. The corresponding solar models are in agreement with the seismic model within $5\ 10^{-3}$.

Alternative models with a mass loss during the first stage of solar evolution have also been considered in relation with the lithium depletion. There is no reliable theory of mass loss along the solar evolution, but some attempts have been made to evaluate its effect on lithium abundance and on solar oscillation frequencies (Guzik & Cox 1995, Morel *et al.* 1997). The lithium abundance at solar age is significantly decreased if mass loss occurs, but not in a sufficient way to account for the observations. Table 2 shows the characteristics of such a model. It appears that such models with decreased lithium abundance are rather close to the seismic models if a mass loss of about 0.1 $M_\odot$ occurs on a time-scale smaller than 0.2 Gy.

4. Helioseismic Constraints

The solar models have to be compared with helioseismic observations. Given the structure of the model it is straightforward to compute the theoretical frequencies of the model, assuming that the oscillations are small perturbations around the equilibrium structure and that they behave adiabatically, i.e. δp is related to $\delta \rho$ through the adiabatic index Γ_1.

There are two approaches to understand the solar interior structure from

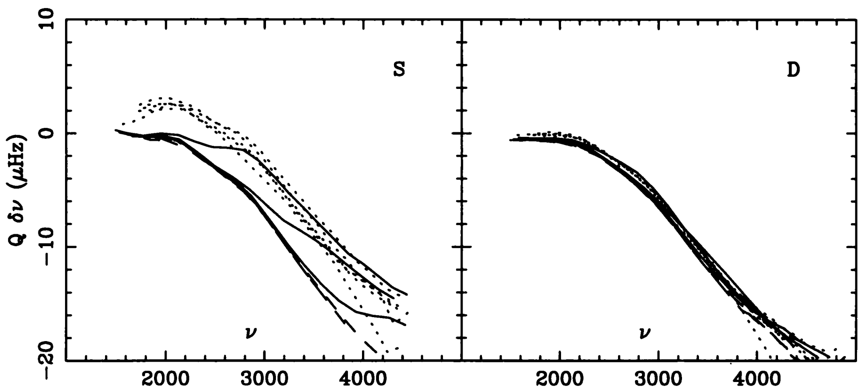

Figure 2. Scaled frequencies differences $Q\delta\nu$, in μHz, between GONG observations and models with and without helium settling as a function of the frequency, for different degrees. The points corresponding to modes of a given degree are linked by continuous curves: ℓ= 2, 3, 4, 5, 10, 20 dotted line; ℓ = 30, 40, 50 full line; ℓ = 70, 100, 120, 140 dashed line. The scaling factor Q is the ratio of the energy of the mode n, ℓ to the energy of the radial mode with the same frequency and the same surface amplitude.

the solar oscillation data: the forward method and the inverse method. This paper is concerned only by the first one. Basu (these proceedings) describes how to obtain a seismic model by inverse method.

The forward method consists in a direct comparison between the observed frequencies and theoretical frequencies of a model evolved with a set of constitutive physics. Since low degree modes are sensitive to the entire solar structure while the highest degree modes probe only the solar surface, the differences between observed and computed frequencies for modes of varying degrees are used as guides to the differences between the model and the Sun, thus as guides to the deficiencies of the input physics. Then new models and frequencies with improved physics are computed, hopefully improving the agreement and informing on the sensitivities of the model structure and frequencies to the input physics. This method has been used to validate improvements in the equation of state and opacities as well as to emphasize the importance of settling of helium and heavy elements in solar modelling.

The actual scaled differences between the observed p modes frequencies, here the initial results of the GONG network (Harvey *et al.*, 1996), and the theoretical frequencies are very small, less than 20 μHz (see Figure 2), i.e. an agreement of a few 10^{-3}. It has been shown that the smooth frequency dependence which appears in the differences is due to problems with the model localized at the surface. An arbitrary increase of the surface opacities may reduce the difference between computed and observed frequencies by decreasing strongly the "slope" in Figure 2 (Christensen-Dalsgaard 1990,

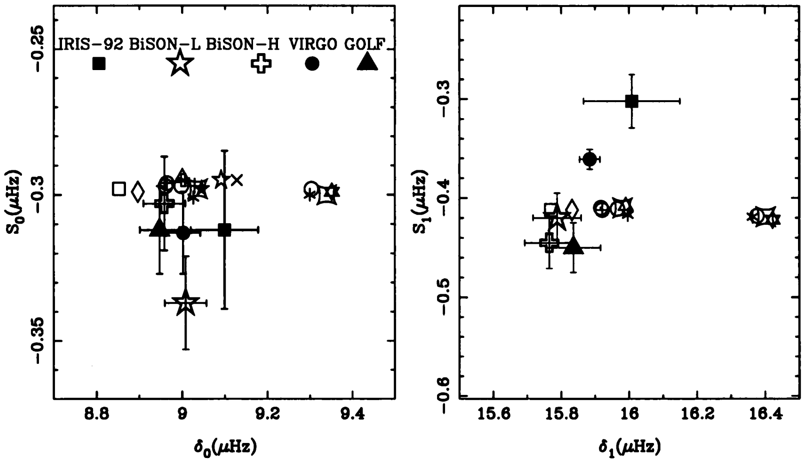

Figure 3. Fit of the frequencies differences $\delta\nu_{n,\ell} = \nu_{n+1,\ell} - \nu_{n,\ell+2}$ for $\ell = 0$ (left) and $\ell = 1$ (right), for recent observations with their error bars and for CESAM solar models (see text). IRIS data are 1992 data. For BiSON L and H refer respectively to low and high solar activity data. The smaller error bars for the VIRGO data are probably due to the fact that the frequencies of modes of degree $\ell = 3$ are determined in an easiest way by the LOI experiment than in full disk observations.

Turck-Chièze & Lopes 1993, Guzik *et al.* 1996). This "slope" is also very sensitive to different treatments of the convection in the solar near surface layers, which is usually described by the mixing length theory. Namely the Canuto-Mazitelli treatment of the convective flux (Basu & Antia 1994) or the inclusion of turbulent pressure (Kosovichev 1995) or a treatment of the convection zone based on results of hydrodynamical simulations (Rosenthal *et al.* 1995) decreases much the differences seen in Figure 2. The contribution of the outer layers to the solar frequencies and their capacity of diagnosing the structure of these layers have been much analysed (e.g. Pérez Hernández & Christensen-Dalsgaard 1994, Brodsky & Vorontsov 1988).

For models with diffusion the dependence of the relative frequencies differences shown in Figure 2 on the degree of the oscillation mode is very small. Nevertheless when subtracting the small frequency dependence it has been shown that the dispersion with degree is of order 1 μHz (Christensen-Dalsgaard *et al.* 1996) with a small separation between the modes which are reflected above or below the base of the convection zone. This is an indication of some problem in the model at these layers that is quantified by seismic inversions (see Basu 1997). The effect of uncertainties on the diffusion coefficients has been discussed, e.g. by Gabriel & Carlier, who have shown that a slight increase of these coefficients cannot be distinguished from a slight increase in opacities, both improving the agreement between observed and theoretical oscillations frequencies.

A strong constraint on the structure of the solar core is provided by low degree p modes frequencies. The small separations between frequencies of modes of consecutive radial order n and of degrees ℓ and $\ell + 2$ are predominantly determined by the conditions in the solar core as shown by the asymptotic analysis

$$\delta\nu_{n\ell} = \nu_{n+1,\ell} - \nu_{n,\ell+2} \approx \frac{2(2\ell+3)}{n+\ell/2+\epsilon}\int_0^R (-\frac{dc}{dr})\frac{dr}{r}$$

where c is the sound speed. The variation of this quantity for radial order high enough can be roughly approximated as:

$$\delta\nu_{n\ell} \approx \delta_\ell + S_\ell(n - n_0).$$

Figure 3 shows the values of δ_ℓ and S_l for $\ell = 0$ and $\ell = 1$ corresponding to solar models including or not the microscopic diffusion (models of Table 2 and from Morel *et al.* (1997)) derived by least square analysis for $n_0 = 21$ and for modes such that $15 \leq n \leq 26$. Results obtained from the same modes for the recent observations by the ground based networks BiSON (Elsworth *et al.*, 1994) and IRIS (Gelly *et al.*, 1997) and by the spatial experiments GOLF (Grec *et al.*, 1997) and VIRGO (Fröhlich *et al.* 1997a-b) are reported with their observational uncertainties. Combining all these data we obtain mean values $\delta_0 = (9.01 \pm 0.05)\mu Hz$ and $\delta_1 = (15.90 \pm 0.08)\mu Hz$, which have been reported in Table 2 and are useful to constrain the theoretical models. In Figure 3 the points corresponding to models with (on the left) and without microscopic diffusion (on the right) are well separated and there is a better agreement with the observations for the models including microscopic diffusion. Note that the "slope" S_ℓ which characterizes the variation of the small separation $\delta\nu_{n\ell}$ with the radial order is the same for the two sets of models.

These constraints provided by low degree p mode have to be satisfied by any solar model particularly those proposed to solve the neutrino problem. Up to now it has revealed to rule out most of the proposed models (see Section 5).

5. The Solar Neutrinos

Another way to look at the core of the Sun and to test nuclear physics involved in stellar main sequence phase is to measure the solar neutrinos. The neutrino fluxes, produced by nuclear reactions inside the Sun, are measured by 4 experiments. The Chlorine experiment (Davis, 1993) is mainly sensitive to ^{8}B and ^{7}Be high energy neutrinos, which depend much of the temperature of the solar core while the Kamiokande water Cerenkov experiment (Fukuda *et al.*, 1996) measures only the ^{8}B neutrinos. The two Gallium experiments GALLEX (Hampel *et al.*, 1996) and SAGE (Abdurashitov *et al.*, 1994) are sensitive to all neutrinos, including low energy pp neutrinos which number is strongly related to the solar luminosity.

The comparisons with theoretical predictions soon revealed a strong deficit of neutrinos measured by the Chlorine experiment and confirmed by the other ones (see Table 2). Consequently much work has been done to modify the physical inputs, specially the nuclear reaction parameters and electron screening for nuclear reactions have been fully reexamined with consequences on solar modelling and neutrino predictions (e.g. Bahcall & Pinsonneault 1992 1995, Turck-Chièze *et al.* 1993). Most of the reactions of the p-p chain which are of importance in the generation of energy are known at a level of 5%. Due to the calibration process to the actual solar luminosity, the structure of the model is not much sensitive to the exact specification of nuclear parameters. On the contrary the reaction rates values are important for the predicted neutrino fluxes and the uncertainties of some of them can be as large as up to 30%: this has been used by Dzitko *et al.* (1995) to obtain a solar model with low ^{8}B and ^{7}Be neutrino fluxes, but as expected with a pp neutrino flux larger than observed by Gallium experiments.

In the frame of the standard description of the solar model given in this review and as far as the more realistic microphysics is used to describe the Sun's interior, each different neutrino experiment has revealed to be incompatible with the others. An alternative explanation which accounts for the different neutrino measurements lies in the physics of the neutrino itself, which, if it has a mass, may oscillate between different states (Mikheyev & Smirnov 1986, Wolfenstein 1978).

As the neutrino fluxes depend much on the core temperature (e.g. Bahcall 1989, Bahcall & Ulmer 1996), many efforts have been developed to explain the neutrinos deficit by lowering the Sun's central temperature and hence to add new physics in the solar models. Different processes have been proposed like turbulent mixing in the core (Schatzman *et al.*, 1981) or energy transport through hypothetical weakly interactive particles named WIMPS (Faulkner *et al.*, 1986). It results respectively too high or too low values of δ_ℓ related to the small low degree p modes differences of frequencies (Provost, 1984; Elsworth *et al.*, 1990). A possible diffusion near the solar core due to stochastic gravity waves has been also investigated (Schatzman & Montalban 1995) and schematized (Morel & Schatzman 1996). More recently Cumming and Haxton (1996) proposed a model with a transport of ^{3}He which results in mixing the Sun on time scales characteristic of ^{3}He equilibrium. All these processes succeed to lower the neutrino fluxes but the corresponding solar models do not satisfy the low degree p modes constraints.

In fact the question is what is the real constraint on the central core provided by helioseismology. The oscillations are mainly determined by the mechanical properties of the Sun through the sound speed and density stratification. Thus they constrain the ratio of the temperature T to the mean molecular weight μ but not each separately. On the contrary the neutrinos fluxes depend crucially from the temperature and the chemical composition independently. Models in hydrostatic equilibrium having the same sound speed as the seismic model have been constructed and the possible range of temperature and neutrino fluxes consistent with helioseismic results has been explored (Dziembowski *et al.* 1994, Antia & Chitre 1995 1997, Shibahashi & Takata 1996 1997, Roxburgh 1997). These so-called "secondary" inversions provide the temperature and helium abundance profiles (see Basu 1997). Antia & Chitre have shown that the reduction of the neutrino fluxes to their observed values requires large unrealistic reduction of opacities. Shibahashi & Takata obtain low boron and berylium neutrino fluxes but their models do not have the solar luminosity. Roxburgh explores the predicted values of neutrino fluxes taking arbitrary profiles of hydrogen abundance and varying the nuclear cross sections well beyond the current estimates. He recovers the observed values, but his approach do not attempt to give a coherent picture of the solar structure and evolution and supposes a slow diffusive mixing and changes of the opacities or an extra contribution to the energy transport across the Sun.

In conclusion all these attempts are not able to reproduce simultaneously the neutrinos fluxes measured by the different experiments and there is no theoretical solar models compatible both with helioseismic and neutrino measurements.

6. Conclusion

Theoretical solar models are in good agreement with the helioseismological constraints, except for the surface layers for which the complexity of the real Sun is not included in the models. There are also significant differences between the Sun and the models specially at the base of the convection zone that remain to be removed. At the present moment no consistent solar model satisfies the constraints provided by the helioseismology and by the measured neutrino fluxes.

To obtain solar models closer of the seismic model inferred from helioseismology, one has specially to improve the description of the physics, which determines the variation of the chemical abundances along the radius, like the treatment of the diffusion and the mixing processes induced by the rotation or by internal gravity waves. This requires to take into account the detailed chemical abundances, which largely influence the opacity. For that purpose one needs opacity tables for a large variety of chemical mixtures, which are not yet available. Another important improvement of the solar model relies in a better description of the structure of the outer layers which are dominated by convection.

The accuracy of helioseismic data which increases with the arrival of the measurements by the spatial experiments on board SOHO may lead to a detection of asphericity and of macroscopic motions and one needs to develop consistent theoretical solar models including rotation and taking fully into account transport of material and of angular momentum, in agreement with the rotation observed inside the Sun. The detection of some solar gravity modes would put strong constraint on the deeper solar interior and increase greatly our knowledge of the solar core.

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