THE ABUNDANCES OF He, N, Ne, Ar AND C1

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### 1. INTRODUCTION

Various abundance problems have been well reviewed by the two previous papers. The study of the compositions of planetary nebulae has been quite fruitful. Nebulae can be used to study elements which are difficult to study in stars. In particular, it has been shown that planetaries can contain matter which has undergone nuclear processing, which in turn can enhance the abundances of some elements, notably He and N. In addition, planetaries can be used to study composition gradients in the galaxy. Of particular relevance to these topics are papers by Peimbert and Torres-Peimbert (1971) (hereafter referred to as PTP) and Boeshaar (1975) on enhancement of N/O; by Kaler (1973) on He/H ratios; by Barker (1976) on He and N; by D'Odorico, Peimbert and Sabbadin (1976), Aller (1976) and Torres-Peimbert and Peimbert (1977) (hereafter referred to as TPP) on abundance gradients.

It is the purpose of this paper to examine various aspects of the abundances of the elements He, N, Ne, Ar and Cl. Results from forthcoming papers (Kaler, in preparation) will be summarized. Broadly, there are two parts to this work. First is the examination of N/O and He/H ratios which show variations among nebulae. Second is the study of the element ratios Ne/O, Ar/O, C1/O for which no, or infrequent, variation would be expected. The unifying theme in this discussion is the examination of various ion ratios in planetaries as a function of nebular excitation or central star temperature. Through this means statements can be made regarding the kind of nebulae in which nitrogen enhancement occurs, the completeness of ionization of helium, and mean values for Ne/O, Ar/O and C1/O. The method used avoids the problem of unobserved ions, and leads to what appears to be the quite accurate Ne, Ar and C1 abundances.

# 2. OBSERVATIONS AND CALCULATIONS

For the studies described above, the line intensities are taken from Kaler's (1976a) catalog, Kaler (unpublished), Lutz (1977), TPP

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Yervant Terzian (ed.), Planetary Nebulae, Observations and Theory, 235-248. All Rights Reserved. Copyright © 1978 by the IAU. and Zipoy (1976). He/H ratios are calculated from the recombination coefficients given by Brocklehurst (1971, 1972), with a small correction for collisional excitation of the  ${}^{3}D$  states of He<sup>+</sup> (Cox and Daltabuit 1971), according to the suggestion by PTP. He/H ratios are calculated only when the He<sup>+</sup> lines are accurately photoelectrically observed.

The other ionic ratios calculated involve forbidden lines. For  $p^2$  and  $p^4$  ions, abundances were calculated from the equations presented by Aller (1956), the transition probabilities given by Garstang (1968) and Nussbaumer (1971) and the target areas given by Seaton (1975) or Osterbrock (1974). For  $p^3$  ions, the solutions given by Saraph and Seaton (1970), Aller, Czyzak, Walker and Krueger (1970), and Czyzak, Krueger and Aller (1970) were employed. For simplicity, the graphs which follow use the Saraph and Seaton (1970) solutions, but final numerical results were figured by using an average of the above 3 references. Differences among them are included in the calculations of error.

The ionic abundances for Ne, Ar and Cl are compared to total oxygen. The abundance of the higher states,  $0^{3+}$ ,  $0^{4+}$ ... is calculated from He<sup>2+</sup>/ He<sup>+</sup> according to Seaton's (1968) suggestion. When He<sup>+</sup> is not accurately observed, this ratio is estimated from (He/H)<sub>0</sub> = 0.166 - 0.00571 R, the helium abundance at distance R(kpc) of the nebula from the galactic center, as taken from TPP and Kaler (unpublished).

A simple two-temperature model is used in the ionic calculations. Temperatures are derived from [NII] and [OIII]. The former are used for  $O^+$  and  $N^+$  calculations, and the latter for all others. If  $T_e[NII]$  is not available, a finding by TPP is adopted in a modified form: if  $I(\lambda 4686)/I(H\beta) > 0.5$ ,  $T_e[NII] = T_e[OIII]/1.4$ , otherwise they are set equal. Electron densities are calculated from [OII], [SII], [CIIII] and [ArIV]. The value used for a given ionic abundance or electron temperature is appropriate to the ionization potential of that ion.

Eight figures are presented in this paper, all showing ion ratios plotted against a measure of nebular excitation. For nebulae for which He<sup>2+</sup> is absent, or only seen in trace amounts, log T\* is used, where T\* is taken from Kaler (1976 and unpublished). Above about 60,000°K where He<sup>2+</sup> is seen, the modified Zanstra method (Harmon and Seaton 1966) would have to be used. For use in the present context there are not enough accurate values available. As a substitute, an excitation parameter called Ex, equal to  $(\text{He}^{2+}/\text{H}^+)/(\text{He}/\text{H})$  is used. The scales are joined such that Ex = 0 at log T\* = 4.75, or T\* ≈ 56000°K, the point at which I( $\lambda$ 4686) first becomes observed. If He/H is not known, (He/H)<sub>0</sub> is adopted from above.

#### NITROGEN

No ionization states of N above  $N^+$  have been used for abundance determination. Because of the similarity of ionization between  $N^+$  and  $O^+$  it has been commonly assumed (see e.g. PTP) that  $N/O = N^+/O^+$ . The problem which this approach is that it implies a correction factor which



Figure 1. Log N<sup>+</sup>/O<sup>+</sup> plotted against (log T<sub>\*</sub> - Ex). Ex is plotted at the top of the figure. Distance from the galactic center is denoted as follows: open symbols, > 10 kpc, closed symbols, < 10 kpc. Circles:  $O^+/O \ge 0.1$ , boxes  $O^+/O < 0.1$ . Vertical lines indicate adjustment for allowed range of T<sub>e</sub>; for high excitation the end of the vertical line denotes N<sup>+</sup>/O<sup>+</sup> for T<sub>e</sub>[OIII] = T<sub>e</sub>[NII]. The size of the symbol is proportional to the weight of the point. The smaller symbols indicate that T<sub>e</sub>[NII] is not measured.

is often quite large, sometimes up to 100. With this equivalency, PTP demonstrated large variations in N/O, and D'Odorico, Peimbert and Sabbadin (1976) and TPP noted the existence of galactic gradients. With a more elaborate method, Boeshaar (1975) suggested a correlation with population type.

The values of log N<sup>+</sup>/0<sup>+</sup> are plotted against log  $T_*$  - Ex in Figure 1. The meaning of the symbols is given in the figure caption. Note the pronounced minimum that occurs at log  $T_* \approx 4.6$  to 4.7. As excitation (and  $T_*$ ) increases, the maximum N<sup>+</sup>/0<sup>+</sup> (and presumably N/O) observed for that excitation increases. That is, at higher excitation, there is a greater likelihood of higher N<sup>+</sup>/0<sup>+</sup>. The effect seems real, and is probably not due to ionization balance, although ionization effects cannot be ruled out. Note that the circles and boxes (see the caption to Figure 1) are rather well mixed; a plot of N<sup>+</sup>/0<sup>+</sup> vs. 0<sup>+</sup>/0 shows no correlation.

The rise of N/O with Ex or  $(T_*)$  could be interpreted as a true abundance effect. A hot nucleus will have had a great deal of its outer envelope stripped away, such as to expose a hot core. Since the ejection process must come close to the hot core, there is a greater chance that CNO products will be mixed into the ejected layers.

The rise in  $N^+/0^+$  for log  $T_* < 4.6$  can probably best be interpreted as an ionization effect. Looking ahead to the next section it is at this point that He is becoming neutral. For these objects most of the ionized oxygen is already in the  $0^+$  state. It would seem that as  $T_*$  decreases,  $0^+$  becomes neutral faster than  $N^+$ , resulting in a higher  $N^+/0^+$  ratio. A full explanation would have to take into account the details of photoionization, recombination, temperature and density fluctuations and charge exchange.

Note also that there seems to be no discrimination between open and filled symbols (objects > 10 kpc or < 10 kpc from the galactic center). Any galactic gradient in N/O is hidden in the large intrinsic variations.

If we average the N/O ratios of the five points at minimum N/O for  $4.6 < \log T_* < 4.73$ , we find  $N/O = 0.12 \pm .02$  (m.e.). This value compares well with Lambert's (1968) solar N/O = 0.145, and appears to be the primordial N/O for planetaries in our general region of the galaxy (all of these 5 nebulae are within 2 kpc of the sun).

4. HELIUM

Barker (1976) indicated that a positive correlation exists between He/H and N/H. If that is true, and especially if the hot central stars produce nebulae enhanced in CNO products, we might expect to see a similar correlation between He/H and  $T_{\star}$ . Such a plot is presented in Figure 2.

In order to remove the galactic gradient  $(He/H)/(He/H)_0$  is plotted against log T<sub>\*</sub> - Ex, such as to show the enhancement in He/H. As we proceed above log T<sub>\*</sub> > 4.7 there seems to be a slight suggestion of an increase of He/H, consistent with N/O, but it is not clear. The apparent rise could as easily be due to incompleteness and observational selection. The matter of He/H is clearly open to question.

The significance of Figure 2 here lies in those objects for which log  $T_* \leq 4.7$ . As we proceed to lower temperatures, He/H rapidly decreases, showing that He is becoming neutral within the H<sup>+</sup> sphere. Two



Figure 2.  $(He/H)/(He/H)_{O}$  plotted against log T<sub>\*</sub> (boxes) or Ex (circles). The sizes of the symbols are proportional to the weights of the points.

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objects, PS-1 and He 2-108, lie above the curve. Either He/H is quite high for these objects or the nebular mass is very low. From Figure 2 it is seen that the star is generally hot enough to ionize He fully for log  $T_* \ge 4.7$  ( $T_* \approx 50000^\circ$ K).

## 5. NEON

It is expected that Ne/O, Ar/O and Cl/O should show no or little variation from one object to another. Ne, Ar and Cl are produced too deeply within the core of the star to be cycled into the ejected envelope. Except for possible galactic variation, mean values of these ratios derived from planetaries would have "cosmic abundance" significance, and could be related to solar values. This and the next two sections makes use of ionization curves (as a function of log  $T_* - E_x$ ), with respect to total oxygen, of Ne<sup>2+</sup>, Ar<sup>2+</sup>, Ar<sup>3+</sup> and Cl<sup>2+</sup> to derive mean abundances. The basic idea is to locate the nebulae for which all or most of an ion is in a given state of ionization, and derives from Kaler (1973). In this paper however, the data are treated in a manner such as to evaluate the errors properly. A summary of the results are presented here; details will be published at a later time.

Figure 3a shows log  $0^{2+}/0$  plotted against log T<sub>\*</sub> - Ex. Note the increase in  $0^{2+}$  to a maximum near log T<sub>\*</sub> = 4.75, followed by a decrease as  $0^{2+}$  becomes ionized to  $0^{3+}$ . From Figure 3a, it is seen that oxygen can be 90% or more in the  $0^{2+}$  state. There is a plateau between 4.64 < log T<sub>\*</sub> < 4.8. The  $0^{2+}/0$  values for which log T<sub>\*</sub> is known and between these limits are averaged to find  $0^{2+}/0 = 0.924 \pm .010$ . In this average the four low points are excluded on the grounds of low accuracy and high, but poorly known, electron density.



Figure 3a (left) and 3b (right) Figure 3a: Log  $O^{2+}/O$  plotted against log  $T_*$  (boxes) - Ex (circles). The crosses denote the Orion Nebula and NGC6523. Figure 3b: Log Ne<sup>2+</sup>/O plotted against log  $T_*$  - Ex, with the same symbolism as for Figure 3a. The filled symbols denote halo objects. The horizontal bars show regions for which averages are taken.

Since Ne<sup>2+</sup> has a range in ionization potentials similar to  $0^{2+}$ , it is argued that 92% of neon will, at maximum, be in the Ne<sup>2+</sup> state. Figure 3b shows log Ne<sup>2+</sup>/O plotted against log  $T_*$  - Ex, where the Ne<sup>2+</sup> abundance is derived from  $\lambda$ 3868[NeIII]. The filled symbols represent the three halo planetaries known: PS 1 in M15 at log  $T_{\star} = 4.43$ , Ha4-1 at Ex = 0.08, and 108-76°1 (see Boeshaar and Bond 1976) at Ex = 0.24. With the exceptions of these three points (discussed below),  $Ne^{2+}/0$ defines an ionization curve which is very similar to that of  $0^{2+}/0$ , with a broad plateau between log  $T_* > 4.6$ , Ex < 0.2. The following discussion thus pertains to objects relatively near the galactic plane. Two averages are taken, one for nebulae for which there exist log  $T_{\star}$  values > 4.6, and another for 0.05 < Ex < 2 (excluding Ha4-1). These regions are denoted by bars in Figure 3b. For both groups,  $Ne^{2+/0} = 0.20 \pm .01$  (m.e.). If 92% of Ne is in the Ne<sup>2+</sup> state, Ne/0 = 0.22  $\pm .01$ . This value compares very well with that of 0.21 found by Acton, Catura and Joki (1975) from solar coronal forbidden lines. The diffuse nebulae plotted in Figure 3b agree well with the ionization curve derived from planetaries, consistent with the agreement between planetary and solar data.

The value found here is less than that of 0.26 derived by TPP, and it agrees well with that published by Aller (1976). A common way of calculating total Ne abundances is to assume that Ne/Ne<sup>2+</sup> =  $0/0^{2+}$ . While this relation will probably be close for the maxima of the ionization curves, it will not be true for individual nebulae, since maximum Ne<sup>2+</sup> occurs at higher T<sub>\*</sub> than does maximum  $0^{2+}$ , thus introducing an error into the extrapolation procedure.

The scatter within the plateau is relatively small. An individual point is expected to have an error of about  $\pm$  20%, which is consistent with errors due to observational scatter, nebular stratification, and errors in the extinction coefficients. There is no need to invoke intrinsic variation, and if such exists it is lost within the random scatter. Plots of Ne<sup>2+</sup>/O for nebulae in the plateau region against distance from the galactic center and against O/H show no significant correlation. Thus, excluding halo objects, Ne/O appears to be constant within about  $\pm$  20%.

The three halo nebulae are anomalous. Ha4-1 is an order of magnitude deficient in neon, while in PS-1 and  $108-76^{\circ}1$  neon appears overabundant. Since PS-1 has such a low T<sub>x</sub>, the anomaly here may be due to ionization. For the other two objects the Ne/O ratio may have galactic significance. They clearly need study in more detail. Unfortunately, Ar and Cl are not yet observed in these objects.

# 6. ARGON

Argon is treated very similarly to neon, but there are some important differences. First there are two ions,  $Ar^{2+}$  and  $Ar^{3+}$ , that are readily available to work with. In addition,  $Ar^{2+}$  often has both nebular (at  $\lambda$ 7135) and auroral (at  $\lambda$ 5191) lines observed. Second, however,



Figures 4a (left) and 4b (right) Figure 4a: Log  $Ar^{2+}/0$  plotted against log  $T_{\star}$  (boxes) - Ex (circles). The sizes of the symbols are proportional to the weights of the points. The arrow indicates log Ar/0. Figure 4b: Log  $Ar^{3+}/0$  plotted against log  $T_{\star}$  = Ex, with the same symbolism as Figure 4a.

is that the span of ionization potentials is such that only the order of half of the Ar is at any time in either  $Ar^{2+}$  or  $Ar^{3+}$  (as can be seen from Ar/O derived below) so that we must use the sum of the two, which cuts the number of available nebulae. The lines are also generally weaker than  $\lambda$ 3868 of [NeIII] so that the resulting errors are greater.

The ionization curve for  $Ar^{2+}/0$  is plotted in Figure 4a. It is similar to those for  $0^{2+}/0$  and  $Ne^{2+}/0$  except that the plateau is absent. The argon is ionized from  $Ar^{+}$  to  $Ar^{2+}$  as  $T_{*}$  increases and is rapidly ionized to  $Ar^{3+}$  with only a small further increase in  $T_{*}$  (or Ex).

Figure 4b shows the ionization curve for  $Ar^{3+}/0$  (from  $\lambda4740$ ). It is complementary to the  $Ar^{2+}/0$  curve, i.e. as  $Ar^{2+}/0$  falls with increasing  $T_x$ ,  $Ar^{3+}/0$  rises. Above Ex = 0.5,  $Ar^{3+}/0$  falls as  $Ar^{3+}$  is ionized to  $Ar^{4+}$ .

The ionization curve for the sum of the two,  $(Ar^{2+} + Ar^{3+})/0$  (or  $Ar^{2,3+}/0$ ) is shown in Figure 5. The curve reaches a peak at Ex ~ 0.1. The points which fall in the range log  $T_* > 4.73$ , Ex < 0.25 are averaged to find  $Ar^{2,3+}/0 = 7.1 \pm .4 \times 10^{-3}$ . The two high points of low weight are not used. The scatter may be accounted for by various sources of error and no intrinsic variations can be seen. If again 92% of the Ar is in  $(Ar^{2+} + Ar^{3+})$ ,  $Ar/0 = 7.7 \pm .5 \times 10^{-3}$ . If this result is compared to the Ne/O found previously, we find Ne/Ar = 29 + 2. This number compares very well with Ne/Ar = 28 found by Marti, Wilkening and Suess (1972) from three gas rich meteorites.

# 7. CHLORINE

For chlorine, only one ion,  $C1^{2+}$  (though  $\lambda$ 5537, 17) is available.



Figure 5. Log  $(Ar^{2+} + Ar^{3+})/0$  plotted against log T<sub>\*</sub> (boxes) - Ex (circles). The sizes of the symbols are proportional to the weights of the points.

Its ionization curve is shown in Figure 6. The ionization potentials for  $C1^{2+}$  and  $Ar^{2+}$  are very similar. Since only  $\approx 60\%$  of Ar is ever in  $Ar^{2+}$  (see Figure 5), the same should be true of  $C1^{2+}$ . Note the close similarity of the slopes of the two ionization curves. Because of these similarities we should be able to find  $\overline{Ar/C1}$  by fitting the curves and reading the difference of the ordinates. The same effect is achieved by assuming  $Ar/C1 = Ar^{2+}/C1^{2+}$  for each nebula, and averaging the results. There are 18 nebulae for which the observations are of sufficient accuracy. The resulting mean  $\overline{Ar/C1} = 19 \pm 3$ . No intrinsic variations are seen. If we adopt the Ar/O value given in the last section, we then find that  $C1/O = 4.1 \pm 0.7 \times 10^{-4}$ . If Ar/C1 for individual nebulae is plotted against log  $T_{\star} - E_{\star}$ , no correlation is seen, reflecting the similarity of the  $Ar^{2+}$ ,  $C1^{2+}$  ionization curves. The value derived above is about twice (but within the error of) that found by comparing Bruhweiler's (1977) C1/H ratio for B stars with Lambert's (1968) solar O/H.



Figure 6. Log  $C1^{2+}/O$  plotted against log T<sub>\*</sub> (boxes) - Ex (circles).

# 8. SUMMARY

The abundance results are summarized in Table 1. The N/O ratio for low excitation (low  $T_*$ ) planetaries is approximately solar, and the ratio can increase by up to an order of magnitude with increasing  $T_*$  or excitation. Apparently, the further the star is stripped down such that a hot core is exposed, the more processed material is likely to be ejected in the envelope. At log  $T_* < 4.6$ , N<sup>+</sup>/O<sup>+</sup> increases with decreasing temperature, which is probably an ionization effect.

At temperatures above log  $T_* = 4.7$ , helium is about fully ionized. There is not a clear rise in He/H with log  $T_* > 4.7$  that can be unambiguously interpreted as an abundance effect; selection effects have not been accounted for.

The Ne/O, Ar/O, Cl/O ratios are probably primordial. Possibly significant variations are seen in Ne/O for halo nebulae. Otherwise, no variation can be seen which cannot be accounted for by various sources of error. It should be noted here that the errors given in Table 1 and in the text are generally the internal mean error. Sources of external error (error in the target areas, etc.) are not taken into account.

If these ratios are primordial they should reflect the ratios predicted by explosive nucleosynthesis calculations. Papers by Truran and Arnett (1970) and Woosley, Arnett and Clayton (1973) predict the abundances of Ar and Cl with respect to Si. If we adopt Lambert and Warner's (1968) solar Si/O ratio, we can find a predicted Ar/O and Cl/O. The average of these predictions is given in the last column of Table 1. The agreement for Ar/O is excellent. The predictions underestimate the Cl abundances however, by a factor which, because of the general agreement between the measurements for planetaries and stars, seems to be greater than the external error.

Ratio	Planetaries	Other Values	Explosive Nucleosynthesis
N/O Ne/O Ar/O Ne/Ar Ar/C1 C1/O	0.12+.02(min.) 0.22+.01 7.7+.5x10 <sup>-3</sup> 29+2 19+3 4.1+.7x10 <sup>-4</sup>	$\begin{array}{c} 0.145(1) \\ 0.21(+.1,5)(2) \\ 28\underline{+}4  (3) \\ 2.1(+2.2, \ -1.0) \times 10^{-4} \\ (4)(1) \end{array}$	$.21(+.1,5) \\ 8\times10^{-3}(5)(6)(7)(1) \\ 85(5)(6) \\ 1.0\times10^{-4}(5)(6)(7) \\ (1)$

Table 1. Summary of abundance calculations. References: (1) Solar (Lambert 1968); (2) Solar coronal forbidden lines (Acton, Catura and Joki 1975); (3) Gas rich meteorites (Marti, Wilkening and Suess 1972); (4) B stars (Bruhweiler 1977); (5) Explosive oxygen burning (Truran and Arnett 1970); (6) Explosive Si-O burning (Woosley, Arnett and Clayton 1973); (7) Solar Si/O (Lambert and Warner 1968). I would like to thank J. W. Truran for helpful discussion, J. R. Dickel for reading the manuscript, and M. Peimbert and F. C. Bruhweiler for providing data in advance of publication. This work was supported by NSF grants MPS 73-05270 and AST 76-20840 to the University of Illinois.

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