

THE ROLE OF REFRACTION IN THE MEASUREMENT OF THREE-DIMENSIONAL
MOVEMENTS BY GEODETIC METHODS

A.H. Dodson,
Department of Civil Engineering,
University of Nottingham,
ENGLAND.

1. INTRODUCTION

The measurement of three-dimensional movements by geodetic techniques involves the determination of spatial co-ordinates by the observation of horizontal theodolite angles and EDM distances, with either vertical angles or spirit levelling providing the height control. For correct interpretation of the results it is essential that estimates of the accuracy of the position determinations are also provided.

Recent legislation, in the United Kingdom (1975 Reservoirs Act) has necessitated the continuous monitoring of water retaining structures. Geodetic techniques have already been used successfully [Ashkenazi, 1975] and further applications of the technique are at present being undertaken. However, one of the major problems in the analysis of the results is the determination of the *a priori* observational accuracies. Field experiments, aimed at providing estimates of the accuracies which can be expected, with various combinations of instruments, have been conducted at Nottingham over several seasons [Ashkenazi and Dodson, 1975] and [Dodson, 1977]. Both the field tests and the monitoring schemes have shown atmospheric refraction to be a major problem, not only because of its effect on the accuracy which can be achieved but also since it makes the estimation of *a priori* accuracies difficult.

This paper briefly describes the field experiments and monitoring schemes (§2 and §3) giving the accuracies which have been achieved. The effect of refraction is discussed in §4 and the conclusions drawn are summarized in §5.

2. FIELD EXPERIMENTS

The basic experiment consisted of observing a movable target from two base stations, the target and base stations forming an approximate

isosceles triangle. Observations were made to the target before and after it had been moved to simulate a deformation. This experiment has been conducted on several sites over different ranges using various combinations of instruments.

For all the tests the base stations have consisted of concrete pillars with forced centring for the instruments. The target consists of a cube corner retro-reflector and separate optical target positioned in an accurately machined mount. This mount provides movement in three mutually perpendicular directions and incorporates vernier scales capable of measuring the movement of the target to 0.1 mm. Since a 'deformation' measurement was conducted in a matter of hours the long term stability of the base pillars was unimportant.

Various combinations of observations were used to test the significance of the inclusion of EDM distances at the varying ranges. Horizontal angles were the mean of three rounds whereas the vertical angles were averaged from two rounds.

The accuracies with which the simulated deformations were monitored, at the various ranges, are given in Table 1 and Table 2. These results are given for movements calculated from two observed horizontal angles, two vertical angles and two distances.

Table 1 shows that with a single-second theodolite the component movements were detected to an accuracy of approximately 1 mm at 50 m range and 3 mm at 350 m range. Over the shorter distance vertical refraction would play little part in the accuracies achieved, however, at 350 m it might be expected that it would be significant. The results do not confirm this expectation, since the δz term is of the same order as the δx and δy terms. This may be due to the particular site where the observations were made, and the fact that only a single-second theodolite was used.

Over the much larger range of 900 m there is a significant difference between the error in z movement determination and the errors in x and y deformation measurement. This difference was almost certainly due to vertical refraction and its effect has been highlighted by the use of higher precision theodolites. It should be noted that the longer range tests were conducted on a different site from the shorter range tests, where the line of sight was closer to the ground. Because of the much larger errors in the determination of the vertical movements some further experiments were conducted at this longer range.

Firstly the vertical angle to the target pillar was monitored continuously throughout the day. A variation of up to 47 seconds of arc was measured.

Secondly an auxiliary, fixed, target was set up adjacent to the movable target and the vertical angle to this second target was

RANGE (m) and No. of sets	INSTRUMENTS	MOVEMENT: OBSERVED - ACTUAL		
		δx (mm)	δy (mm)	δz (mm)
50 :- 12 sets	WILD T2 GEODIMETER 6A	1.1	0.8	0.7
350 :- 7 sets	WILD T2 KERN MEKOMETER	4.3	3.2	3.3
350 :- 3 sets	WILD T2 TELLUROMETER MA 100	6.2	3.4	2.8

Short Range Deformation Tests

Mean Experimental Accuracies

Table 1

RANGE (m) and No. of sets	INSTRUMENTS	MOVEMENT: OBSERVED - ACTUAL		
		δx (mm)	δy (mm)	δz (mm)
870 :- 4 sets	WILD T3 HEWLETT-PACKARD 3800B	3.1	2.1	16.4
870 :- 6 sets	KERN DKM3 HEWLETT-PACKARD 3800B	3.4	3.7	10.1
950 :- 13 sets	KERN DKM3 TELLUROMETER MA 100	2.7	1.8	29.6

Long Range Deformation Tests

Mean Experimental Accuracies

Table 2

measured both before and after the movement of the deformation target. Thus the refraction effects, on the vertical angle to the deformation target, could be eliminated by referring all the vertical angle measurements to the fixed auxiliary target. Table 3 gives the results obtained by correcting the observations for vertical refraction in the manner described. It can be seen that a significant improvement has been obtained, and that the accuracy of the determination of vertical movement has been brought nearer to that achieved for horizontal movement.

RANGE	INSTRUMENTS	MOVEMENT: OBSERVED-ACTUAL					
		BEFORE CORRECTION			AFTER CORRECTION		
		δx	δy	δz	δx	δy	δz
950 m	KERN DKM3 TELLUROMETER	3.6	2.1	36.3	3.5	1.8	9.4
:- 5 sets	MA 100						

Long Range Deformation Tests

Refraction Correction

Table 3

3. DEFORMATION MONITORING SCHEMES

A deformation monitoring scheme has already been successfully employed [Ashkenazi, 1975] and two further schemes, involving contracts with Nottingham University, are being undertaken at present. These schemes consist of the establishment of a control network, observed by theodolite and EDM instruments, which is to be adjusted by a rigorous least squares procedure. Vertical control is provided by reciprocal (though not simultaneous) vertical angle measurement, with the possibility of some spirit levelling. Numerous detail points on the structures in question can be monitored from any two control stations. Some detail points will not be easily accessible and hence spirit levelling will be impracticable. The accuracies required from these schemes vary from 1-2 mm to 10 mm and the ranges over which the observations are made are between 100 m and 2 km.

The accuracies achieved in the first scheme, where the instruments used were a WILD T3 theodolite and a TELLUROMETER MA 100, were of the order of 3 mm for absolute horizontal position [Ashkenazi, 1975]. However, tests have indicated that an a priori accuracy, for height control, of approximately 10 mm per km can be expected for a line levelled by reciprocal vertical angle measurement. This would lead to

à posteriori accuracies of absolute vertical position of the order of 5 mm. Nevertheless even this accuracy can be drastically affected if there exist either unusually poor conditions for observation or lines which are particularly susceptible to refraction errors. Vertical positional errors of the order of 50 mm have been experienced on certain occasions. The possible accuracy of the vertical control on these schemes is thus much less than that of the horizontal control.

The accuracy of the determination of the detail point positions obviously depends largely on the accuracy of the control stations used. Provided the range from the control stations to the detail points is short (< 200 m) then, unless very unusual conditions exist, the additional error due to the detail point observations will be of the order of 1-3 mm (see §2).

4. REFRACTION

The major influence of refraction on the measurement of three dimensional movements is its effect on the determination of heights by vertical angle observations. §2 described the results obtained in field experiments and it is clear that large errors can arise from one-way vertical angle measurements. §3 explains how, even with reciprocal (though not simultaneous) observations the accuracy of height control is less than that which can be reasonably easily obtained for horizontal position.

The errors in vertical angles described in §2 are of the order of 5 - 10 seconds of arc, in general, and experience has shown that this is also the magnitude of error to be expected from one-way observations in actual monitoring schemes. The use of reciprocal observations, where possible, reduces this error but at a considerable cost in time and manpower. Simultaneous observations would almost certainly reduce the error still further but again at increased cost. The obvious alternative of spirit levelling would provide better precision but involves greatly increased expenditure and, because of the terrain and accessibility of stations, is not always feasible. However, for the more precise surveys, unless vertical refraction errors can be reduced to the order of 1 - 2 seconds of arc, spirit levelling remains essential.

The second, less significant, effect of atmosphere refraction is on the distances measured using EDM instruments. The uncertainties in the refractive index can cause random errors of the order of 5 ppm. However, our experience at Nottingham [Ashkenazi, 1975], [Dodson, 1977], [Ashkenazi and Dodson, 1977] has shown that the random proportional errors, when using instruments such as the TELLUROMETER MA 100 and KERN Mekometer ME 3000, may be estimated at approximately 2 ppm (an average value throughout a network) and that, assuming the instruments used have been properly calibrated, this error does not significantly affect the accuracy of horizontal position determination

provided sufficient horizontal angles are measured. Obviously any systematic refraction effects (due maybe to the determination of the atmospheric parameters) will not be included in this accuracy estimate. Reduction of atmospheric refraction errors in EDM distances to the order of 1 ppm (including any systematic effects) would obviously increase the accuracy of movement determination but more significantly would allow fewer observations to be made (particularly time consuming angular measurements) whilst still maintaining the precision now available.

Whether the reduction of atmospheric refraction errors is achieved through instrument development [Huggett and Slater, 1977], [Tengström, 1977] and [Williams, 1977] or by an improvement in atmospheric modelling techniques [Brunner, 1977], [Brunner and Fraser, 1977], [Felletschin, 1977] and [Maier, 1977] is a matter for discussion but, in either case the improvement would be valuable for the accurate determination of three-dimensional movements.

5. CONCLUSIONS

(a) Field experiments have shown atmospheric refraction to have a significant effect on the determination of the vertical movement of a target at 950 m range.

(b) If corrections are applied for vertical refraction three-dimensional movements can be monitored to about 3 mm in the horizontal plane and 9 mm vertically, at a range of 950 m.

(c) Horizontal position accuracies of 2 - 3 mm have been achieved for a deformation monitoring scheme but atmospheric refraction limits the height control (determined by reciprocal vertical angle measurements) to between 5 and 10 mm generally, with some observations being in error by 50 mm.

(d) Vertical position accuracies could be of a comparable magnitude to the horizontal accuracies if vertical refraction effects could be reduced by means of either dispersometer type instrumentation or improved atmospheric modelling techniques.

(e) At present high accuracy vertical control can only be provided (where possible) by time consuming and thus expensive spirit levelling techniques.

(f) The effect of uncertainties in the refractive index of the atmosphere introduces errors into EDM observations. A reduction in these errors, either by use of multi-wavelength EDM or by atmospheric modelling, would enable higher accuracies to be achieved in horizontal movement determination.

(g) Furthermore, improvements in observational accuracy would

enable economies to be made in the number of observations required to produce a network of a given strength.

(h) High precision monitoring of three-dimensional movements would benefit greatly from the reduction of the errors in observations due to atmospheric refraction.

REFERENCES

- Ashkenazi, V. (1975) *Deformation measurements at the Empingham Dam site*, Proc. F.I.G. Symp., Cracow.
- Ashkenazi, V. and Dodson, A.H. (1975) *Experimental accuracies in the remote measurement of spatial deformations by geodetic methods*, Proc. F.I.G. Symp., Cracow.
- Ashkenazi, V. and Dodson, A.H. (1977) *The calibration and evaluation of EDM instruments*, Proc. XV F.I.G. Congress, Stockholm.
- Brunner, F.K. (1977) *Experimental determination of the co-efficients of refraction from heat flux measurements*, Proc. I.A.G. Symp., Wageningen.
- Brunner, F.K. and Fraser, C.S. (1977) *An atmospheric turbulent transfer model for EDM reduction*, Proc. I.A.G. Symp., Wageningen.
- Dodson, A.H. (1977) *The measurement of spatial displacements by geodetic methods*, Ph.D. Thesis, University of Nottingham.
- Felletschin, V. (1977) *Possibilities for raising the accuracy of light and microwave instruments*, Proc. I.A.G. Symp., Wageningen.
- Huggett, G.R. and Slater, L.E. (1977) *Recent advances in multi-wavelength distance measurement*, Proc. I.A.G. Symp., Wageningen.
- Maier, U. (1977) *Distance measurements over the Rhine valley*, Proc. I.A.G. Symp., Wageningen.
- Tengström, E. (1977) *Some absolute tests of the results of IDM measurements in the field with a description of the formulae used in the tests*, Proc. I.A.G. Symp., Wageningen.
- Williams, D.C. (1977) *First field tests of an angular dual wavelength instrument*, Proc. I.A.G. Symp., Wageningen.

DISCUSSION

This was mainly a dialogue between Dr Dodson and professor Kukkamäki about the possibility of using levelling instead of vertical angles to determine accurate heights. Dodson agreed with Kukkamäki's statement that levelling is desirable, but pointed out that the costs for such a survey, and the difficulties to carry it out, were so great that he had been forced to rely upon trigonometrical levelling, the accuracy of which usually satisfied his customers' requirements. Moreover, by correcting for refraction by means of methods which are under development, the accuracy will probably be increased to satisfy all needs in the future. Dr Holdahl interfered, as regards the costs of levelling;

S.R. Holdahl: Another advantage of leveling, in locations where you can use it, is that a detailed profile of elevation can be established at low cost. When leveling 1 km, it is customary to determine a height at each of 10 or more turning points, one for each setup of the instrument. If the turning points (points on which the rods are set) are solid monuments, the observational cost of determining heights at the ten points is no different from the cost of determining the height difference between the first and last points. The cost of additional monumentation is an expenditure made only once, at the time of the first survey.