ROTATIONAL CORRELATION IN BINARY STARS

RAPHAEL STEINITZ and DIANE M. PYPER

Tel Aviv University, Tel Aviv, Israel

Abstract. A method for testing the possible correlation between axial rotations of pairs of stars is developed. The test is applied to a sample of visual binaries. It is concluded that some kind of coupling between the spins of components of visual binaries does in fact exist.

1. Introduction

The study of the distribution of orbital and spin angular momenta in binary and multiple star systems is vital to the development of a theory concerning the origin and evolution of such systems. Some studies of this type have recently been carried out. For instance, Huang and Wade (1966) conclude that the orbital angular momenta of binaries are randomly oriented in our galaxy. Another example is Slettebak's (1963) determination that no significant differences between the mean rotational velocities for components of visual binaries and single stars exist. However, these analyses, while they are of prime importance concerning possible theories of origin, are not concerned with coupling between the various angular momenta in a binary system.

In this paper, we develop a theory enabling us to determine the possible existence of coupling between the spin angular momenta in binaries. In Section 5 this theory is applied to a sample of visual binaries. A method for testing similar connections between orbital and spin angular momenta will be developed in a separate paper.

2. Mathematical Theory of Rotational Correlation

Let the observed rotational velocities of the primary and secondary stars in a binary system be u_1 and u_2. We wish to express the bivariate distribution function, $F(u_1, u_2)$, of the observed rotational velocities as a function of the true distribution of rotational velocities (v_1 and v_2) and the distribution of the angle (θ) between the spin axes of the two stars, i.e., as a functional of the distribution function $G(v_1, v_2, \theta)$.

To this end, we define the following quantities:

The primary and secondary stars have spins that are characterized by unit vectors $\mathbf{S}_1$ and $\mathbf{S}_2$, respectively. We choose the coordinate system with the z-axis aligned along $\mathbf{S}_1$ and the (x, z) plane containing $\mathbf{S}_2$. As stated above, θ is the angle between $\mathbf{S}_1$ and $\mathbf{S}_2$. The line of sight is defined by the unit vector $\mathbf{A}$; the angles between $\mathbf{S}_1$ and $\mathbf{A}$ and between $\mathbf{S}_2$ and $\mathbf{A}$ are i_1 and i_2, respectively (Figure 1). Now, clearly,

$$u_1 = v_1 \sin i_1$$

and

$$u_2 = v_2 \sin i_2 .$$

Finally, let ϕ be the angle between the (x, z) plane and the plane containing $\mathbf{S}_1$ and $\mathbf{A}$.

A. Slettebak (ed.), Stellar Rotation, 165–177. All Rights Reserved

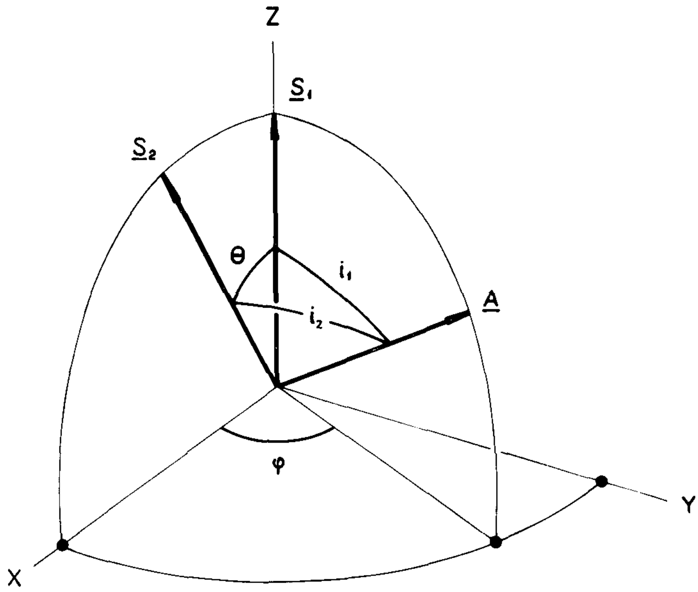

Fig. 1. Geometry of the problem.

For the sake of brevity, we use the notation $E[q_1, q_2, \ldots q_r]$ to denote the event that $q_j \in [q_j, q_j + \mathrm{d}q_j]$, for $j = 1, 2, \ldots, r$; and the notation $P(E[q_1, q_2, \ldots, q_r])\,\mathrm{d}q_1\,\mathrm{d}q_2 \ldots \mathrm{d}q_r$ for the probability of E. Now, the probability of $E[v_1, v_2, \theta]$ is given by $G(v_1, v_2, \theta)\sin\theta\,\mathrm{d}v_1\,\mathrm{d}v_2\,\mathrm{d}\theta$. The following particular cases should be considered:

(a) The angular distribution does not depend on θ. In that case,

$$G(v_1, v_2, \theta) = H(v_1, v_2).$$

(b) There is a dependence on θ, but that distribution is independent of the distribution of velocities. Then,

$$G(v_1, v_2, \theta) = H(v_1, v_2)\,g(\theta).$$

(c) The distribution of v_1 and v_2 are independent of each other, so that

$$G(v_1, v_2, \theta) = f_1(v_1)\,f_2(v_2)\,g(\theta).$$

Let the probability of $E[v_1, v_2, \theta; i_1, i_2]$ be given by

$$D(v_1, v_2, \theta; i_1, i_2)\,\mathrm{d}v_1\,\mathrm{d}v_2\,\mathrm{d}\theta\,\mathrm{d}i_1\,\mathrm{d}i_2.$$

This probability cannot be readily expressed in terms of θ, i_1 and i_2, since they are not wholly independent; the specification of the values of two of them will, in fact, limit the variation of the third. In order to determine $D(v_1, v_2, \theta; i_1, i_2)$, we shall first consider $E[v_1, v_2, \theta; i_1, \phi]$, since i_1, θ and ϕ are independent variables. Its probability is given by

$$P(E[v_1, v_2, \theta; i_1, \phi])\,\mathrm{d}v_1\,\mathrm{d}v_2\,\mathrm{d}\theta\,\mathrm{d}i_1\,\mathrm{d}\phi$$
$$= NG(v_1, v_2, \theta)\sin\theta\sin i_1\,\mathrm{d}v_1\,\mathrm{d}v_2\,\mathrm{d}\theta\,\mathrm{d}i_1\mathrm{d}\phi$$

where N is a normalizing factor.

In general (e.g., see Trumpler and Weaver, 1953), the transformation of probability densities from one set of variables $(q_1, \ldots, q_r)$ to a second set $(Q_1, \ldots, Q_r)$ is given by

$$P(E[Q_1, \ldots, Q_r]) = P(E[q_1, \ldots, q_r]) \cdot \left| J\left(\frac{q_1, \ldots, q_r}{Q_1, \ldots, Q_r}\right) \right|,$$

where

$$J\left(\frac{q_1, \ldots, q_r}{Q_1, \ldots, Q_r}\right)$$

is the determinant of the Jacobian matrix of the partial derivatives of $(q_1, \ldots, q_r)$ with respect to $(Q_1, \ldots, Q_r)$. Therefore,

$$P(E[v_1, v_2, \theta; i_1, i_2]) = P(E[v_1, v_2, \theta; i_1, \varphi]) \cdot \left| J\left(\frac{v_1, v_2, \theta, i_1, \varphi}{v_1, v_2, \theta, i_1, i_2}\right) \right|. \tag{1}$$

Now, as can be easily seen,

$$J\left(\frac{v_1, v_2, \theta, i_1, \varphi}{v_1, v_2, \theta, i_1, i_2}\right) = J\left(\frac{v_1, v_2}{v_1, v_2}\right) J\left(\frac{\theta, i_1, \varphi}{\theta, i_1, i_2}\right)$$

and

$$J\left(\frac{\theta, i_1, \varphi}{\theta, i_1, i_2}\right) = \frac{\partial \varphi}{\partial i_2}.$$

From the geometry (Figure 1), it follows that

$$\cos i_2 = \cos\theta \cos i_1 + \sin\theta \sin i_1 \cos\phi, \tag{2}$$

so that

$$\frac{\partial \varphi}{\partial i_2} = \frac{\sin i_2}{(Q(\theta, i_1, i_2))^{1/2}}, \tag{3}$$

where, in general,

$$Q(\alpha, \beta, \gamma) = 1 - \cos^2\alpha - \cos^2\beta - \cos^2\gamma + 2\cos\alpha \cos\beta \cos\gamma. \tag{4}$$

Therefore, from (1) and (3),

$$P(E[v_1, v_2, \theta; i_1, i_2]) = N \cdot G(v_1, v_2, \theta) \frac{\sin\theta \sin i_1 \sin i_2}{|(Q(\theta, i_1, i_2))^{1/2}|}. \tag{5}$$

In passing, it should be noted that except for the dependence on θ through $G(v_1, v_2, \theta)$ the expression for $P(E[v_1, v_2, \theta; i_1, i_2])$ is completely symmetric in θ, i_1 and i_2, as it should be.

We now express $P(E[v_1, v_2, \theta; u_1, u_2])$ in terms of $E[v_1, v_2, \theta; i_1, i_2]$:

$$P(E[v_1, v_2, \theta; u_1, u_2]) = P(E[v_1, v_2, \theta; i_1, i_2]) \cdot \left| J\left(\frac{v_1, v_2, \theta, i_1, i_2}{v_1, v_2, \theta, u_1, u_2}\right) \right|.$$

When evaluated, it is found that

$$J\left(\frac{v_1, v_2, \theta, i_1, i_2}{v_1, v_2, \theta, u_1, u_2}\right) = \frac{1}{[(v_1^2 - u_1^2)(v_2^2 - u_2^2)]^{1/2}}. \tag{7}$$

https://doi.org/10.1017/S0252921100027160 Published online by Cambridge University Press

In order to obtain $F(u_1, u_2)$, it is necessary to integrate (6) over v_1, v_2 and θ. Clearly, the limits of integration for v_1 and v_2 are from u_1 to ∞ and from u_2 to ∞, respectively. It can readily be verified that $|i_1 - i_2| \leqslant \theta \leqslant i_1 + i_2$ for all values of i_1 and i_2 (Figure 1). Since we have transformed the set of variables $(v_1, v_2, \theta; i_1, i_2) \to (v_1, v_2, \theta; u_1, u_2)$, i_1 and i_2 should be expressed in terms of the new variables. The limits of integration over θ are therefore given by

$$\begin{aligned} \Theta_- &= |\arcsin(u_1/v_1) - \arcsin(u_2/v_2)| \\ \Theta_+ &= \arcsin(u_1/v_1) + \arcsin(u_2/v_2). \end{aligned} \tag{8}$$

Due to the above transformation, it is also necessary to rewrite $Q(\theta, i_1, i_2)$ as follows:

$$\begin{aligned} Q(\theta, i_1, i_2) = R\left(\theta, \frac{u_1}{v_1}, \frac{u_2}{v_2}\right) &\equiv \sin^2\theta + \left(\frac{u_1}{v_1}\right)^2 \\ &+ \left(\frac{u_2}{v_2}\right)^2 + 2\left\{\cos\theta\left(\left[1 - \left(\frac{u_1}{v_1}\right)^2\right]\left[1 - \left(\frac{u_2}{v_2}\right)^2\right]\right)^{1/2} - 1\right\}. \end{aligned} \tag{9}$$

After we make the appropriate substitutions, we finally obtain:

$$\begin{aligned} \frac{F(u_1, u_2)}{u_1 u_2} = N \int_{u_1}^{\infty} \int_{u_2}^{\infty} \int_{\Theta_-}^{\Theta_+} \frac{G(v_1, v_2, \theta)}{v_1 v_2} \\ \times \frac{\sin\theta \, d\theta \, dv_1 \, dv_2}{\left[R\left(\theta, \frac{u_1}{v_1}, \frac{u_2}{v_2}\right)\right]^{1/2} [(v_1^2 - u_1^2)(v_2^2 - u_2^2)]^{1/2}}. \end{aligned} \tag{10}$$

When we consider case (b) above, (10) becomes:

$$\begin{aligned} \frac{F(u_1, u_2)}{u_1 u_2} = N \int_{u_1}^{\infty} \int_{u_2}^{\infty} \frac{H(v_1, v_2)}{v_1 v_2} \\ \times \left[\int_{\Theta_-}^{\Theta_+} \frac{g(\theta) \sin\theta \, d\theta}{\left[R\left(\theta, \frac{u_1}{v_1}, \frac{u_2}{v_2}\right)\right]^{1/2}}\right] \frac{dv_1 \, dv_2}{[(v_1^2 - u_1^2)(v_2^2 - u_2^2)]^{1/2}}; \end{aligned} \tag{11}$$

while in case (c) we have:

$$\begin{aligned} \frac{F(u_1, u_2)}{u_1 u_2} = N \int_{u_1}^{\infty} \int_{u_2}^{\infty} \frac{f_1(v_1)}{v_1 (v_1^2 - u_1^2)^{1/2}} \cdot \frac{f_2(v_2)}{v_2 (v_2^2 - u_2^2)^{1/2}} \\ \times \left[\int_{\Theta_-}^{\Theta_+} \frac{g(\theta) \sin\theta \, d\theta}{\left[R\left(\theta, \frac{u_1}{v_1}, \frac{u_2}{v_2}\right)\right]^{1/2}}\right] dv_1 \, dv_2; \end{aligned} \tag{12}$$

and under the special circumstance that $g(\theta) \equiv 1$ (case (a) above), we see that

$$\int_{\Theta_-}^{\Theta_+} \frac{\sin\theta\, d\theta}{\left[R\left(\theta, \frac{u_1}{v_1}, \frac{u_2}{v_2}\right)\right]^{1/2}} = \left[-\arcsin\left\{\frac{v_1 v_2}{u_1 u_2} \times \left[\cos\theta - \left(1 - \left(\frac{u_1}{v_1}\right)^2\right)^{1/2} \left(1 - \left(\frac{u_1}{v_2}\right)^2\right)^{1/2}\right]\right\}\right]_{\Theta_-}^{\Theta_+} = \pi . \tag{13}$$

It is therefore evident that $N = \pi$.

Suppose we consider pairs of stars formed at random from a sample of single stars; then

$$\frac{F(u_1, u_2)}{u_1 u_2} = \int_{u_1}^{\infty} \frac{f_1(v_1)\, dv_1}{v_1 (v_1^2 - u_1^2)^{1/2}} \cdot \int_{u_2}^{\infty} \frac{f_2(v_2)\, dv_2}{v_2 (v_2^2 - u_2^2)^{1/2}}, \tag{14}$$

or,

$$F(u_1, u_2) = F_1(u_1) \cdot F_2(u_2), \tag{15}$$

as is to be expected; i.e., the observed bivariate distribution function is the product of the observed univariate distributions.

If members of binary systems were rotationally uncorrelated, it would be found that $F(u_1, u_2)$ would be represented by (15). However, if this is not found to be the case, it is, at least *a priori*, impossible to distinguish between the most general case $G(v_1, v_2, \theta)$ and case (b), where $G(v_1, v_2, \theta) = H(v_1, v_2)\, g(\theta)$.

3. Choice of a Relevant Sample

The choice of a relevant sample for testing the idea of rotational correlation will be determined by physical factors as well as considerations of observational selection effects.

Some very short-period spectroscopic binaries are interpreted as having synchronous rotation (see Struve, 1950), i.e. the rotational period of the components is equal to their orbital period. This is explained in terms of tidal forces, which are supposed to have brought about the synchronization. In some other close binaries, the period of the axial rotation of at least the primary is undoubtedly shorter than the orbital period of the pair. It has been suggested that this could be due to mass transfer between the components (e.g., see Batten, 1967). In either case, it would seem that the initial conditions which existed during the first phases of the evolution of the system no longer prevail.

If, on the other hand, visual binaries are not the result of the disruption of close systems (Chandrasekhar, 1944; Ambartsumian, 1937), and both components are on the main sequence, we cannot see any reason why such systems should not reflect now the conditions which prevailed during their formation period.

Observational selection effects should also be considered. Spectra of both components of a spectroscopic pair are visible only if their magnitude difference, Δm, is less than about one magnitude (Struve, 1950). This rather severe restriction does not apply in the case of visual binaries. As can be seen from Table I (Bečvář, 1964) the range in Δm for visual binaries is certainly larger than in spectroscopic binaries.

TABLE I
Magnitude difference and separation in visual binaries

Star	m_1	m_2	Sep (″)
66 Ari	6.11	12.2	1.0
β^1 Tuc	4.52	14.0	2.2

The actual value of Δm allowing one to recognize a visual binary as such depends in general on the telescope aperture and the apparent separation of the components. In a survey of rotational velocities, however, Δm is only limited by the instrumentation and by seeing effects, not by the physical characteristics of the pair. Additional selection effects are present when spectroscopic binaries are considered. Accurate values of rotational velocities cannot be measured in double-lined systems if the lines of the two components are too badly blended. There results a bias in favor of systems having large velocity amplitudes; i.e., large masses, short periods, or both. Moreover, this effect becomes larger as the line width increases.

Due to the arguments presented above, it is concluded that a sample of visual binaries would be the most meaningful one as a means of testing for rotational correlation in binaries.

4. The Data

The only large amount of data available to us concerning rotational velocities in visual binaries is a study of 116 pairs by Slettebak (1963).

Since the lower limit for the values of $v \sin i$ in this investigation is 25 km/sec, a meaningful discussion of the slowly-rotating late-type stars is impossible. We therefore have excluded all pairs having components of spectral type F or later. Giants and supergiants were rejected also on the grounds that they presumably no longer possess their original rotational velocities, due to their increase in radius and to possible mass loss.

From the list, therefore, all pairs which satisfy the following conditions were selected:

(a) The spectral types of both components are earlier than A9.

(b) Both components are on the main sequence.

When these criteria are applied, a sample of 50 pairs is obtained. The total range of spectral types is from O8 to A9. Visual binaries having components which are also single-lined spectroscopic binaries are included in the sample, as well as Ap and Am stars.

5. Statistical Analysis

A. OBSERVED VELOCITY CORRELATION

Table II shows the relative frequency distribution function, $F(u_1, u_2)$, of observed rotational velocities of the sample discussed in Section 4. The observed rotational velocities of the primary and secondary are u_1 and u_2, respectively. Due to the smallness of the sample, the velocity range has been divided into 50 km/sec sub-intervals.

TABLE II

Bivariate distribution of rotational velocities in visual binaries, $F(u_1, u_2)$

u_2 \ u_1		50	100	150	200	250	300	$\phi_2(u_2)$	$\bar{u}_1(u_2)$
0									
	0.12	0.02	0.02	0.02			0.02	0.20	85
50									
	0.04	0.10	0.06	0.04				0.24	96
100									
	0.04	0.04	0.04	0.04	0.02	0.02	0.02	0.22	148
150									
	0.02	0.04	0.02	0.04				0.12	108
200									
	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.14	175
250									
				0.04			0.02	0.06	225
300									
							0.02	0.02	325
$\phi_1(u_1)$	0.24	0.22	0.16	0.20	0.04	0.04	0.10		
$\bar{u}_2(u_1)$	79	111	112	155	175	175	195		

The marginal distribution, $\phi_1(u_1)$, gives the distribution of observed rotational velocities of primaries independent of the secondaries. The corresponding marginal distribution for the secondaries is $\phi_2(u_2)$. From these, another array (not shown), $\Phi(u_1, u_2) \equiv \phi_1(u_1) \cdot \phi_2(u_2)$ is derived. This array represents a bivariate distribution of observed velocities of pairs of stars which are formed by matching to each primary a secondary at random, out of the sample of selected binaries. The array $\Phi(u_1, u_2)$ represents, therefore, a distribution of two independent variables. Finally, the array of differences between the true distribution and the artificial one, $\Delta(u_1, u_2) \equiv F(u_1, u_2) - \Phi(u_1, u_2)$ is given in Table III. It should be noted that there is a marked tendency for positive differences to be found along the main diagonal.

According to Trumpler and Weaver (1953) there are various ways of testing correlation between variables in a bivariate distribution. As a first step, the correlation coefficient, r, was computed for $F(u_1, u_2)$. It was found that $r = 0.46$, while the correlation coefficient for the artificial distribution $\Phi(u_1, u_2)$ gives $r = 0.001$, as would be expected.

As a second step, the regression curves $\bar{u}_1(u_2)$ and $\bar{u}_2(u_1)$ were computed. These

https://doi.org/10.1017/S0252921100027160 Published online by Cambridge University Press

TABLE III

Differences: $\Delta(u_1, u_2) = F(u_1, u_2) - \phi_1(u_1) \cdot \phi_2(u_2)$

u_1 \ u_2	50	100	150	200	250	300	
	0.072	−0.024	−0.012	−0.020	−0.008	−0.008	0.000
50	−0.018	0.047	0.022	−0.008	−0.010	−0.010	−0.024
100	−0.013	−0.008	0.005	−0.004	0.011	0.011	−0.002
150	−0.009	0.014	0.001	0.016	−0.005	−0.005	−0.012
200	−0.014	−0.011	−0.002	−0.008	0.014	0.014	0.006
250	−0.014	−0.013	−0.010	0.028	−0.002	−0.002	0.014
300	−0.005	−0.004	−0.003	−0.004	−0.001	−0.001	0.018

curves give the variation of the conditional mean velocity of the primaries as a function of the velocity interval of the secondary and vice-versa. In terms of the velocity intervals, $u_1(i)$ and $u_2(j)$, the regression curves are defined to be:

$$\begin{aligned} \bar{u}_1(u_2(j)) &= \sum_{i=1}^{7} u_1(i)\, F(u_1(i), u_2(j)) / \sum_{i=1}^{7} F(u_1(i), u_2(j)) \\ \bar{u}_2(u_1(i)) &= \sum_{j=1}^{7} u_2(j)\, F(u_1(i), u_2(j)) / \sum_{j=1}^{7} F(u_1(i), u_2(j)). \end{aligned} \tag{16}$$

The regression curves are graphically represented in Figures 2 and 3.

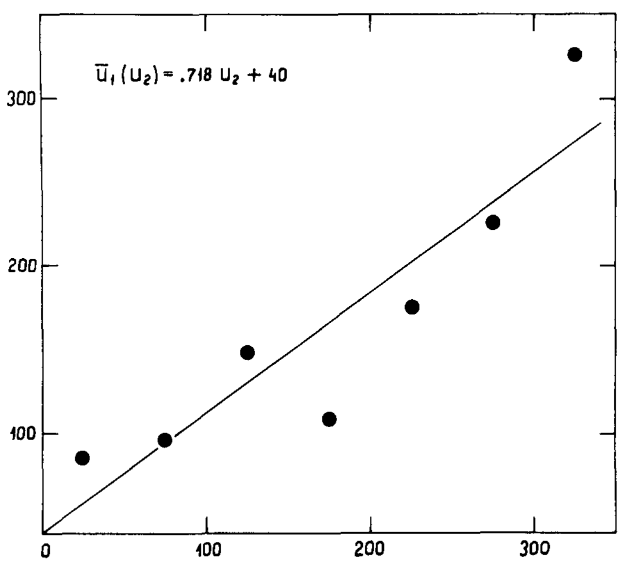

Fig. 2. Regression of the mean rotational velocities of primaries (*ordinate*) as a function of the rotational velocity interval of secondaries (*abscissa*).

https://doi.org/10.1017/S0252921100027160 Published online by Cambridge University Press

It can be seen that definite trends are present, in the sense that the mean observed rotational velocity of one of the components is an increasing function of the rotational velocity of the other. In fact, it appears that the relationship may be linear. A least squares solution for a straight-line fit to each regression curve is also shown.

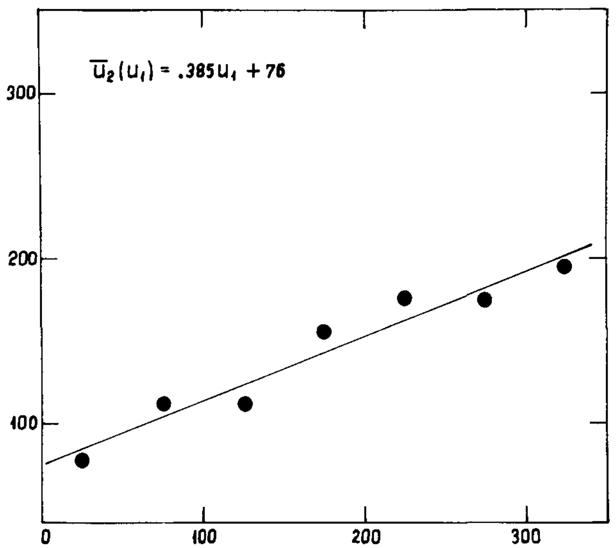

Fig. 3. Regression of the mean rotational velocities of secondaries (*ordinate*) as a function of the rotational velocity interval of primaries (*abscissa*).

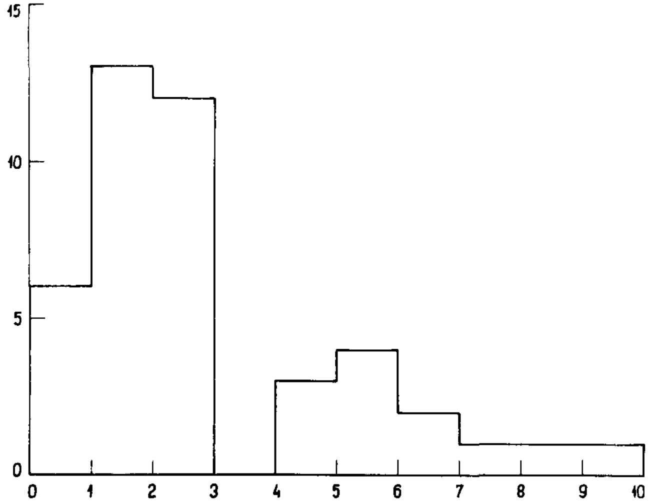

Fig. 4. Distribution of spectral type intervals (differences in subclasses) in the sample of binaries.

B. A SELECTION EFFECT

It appears that the sample selected for the statistical analysis is heavily biased in favor of pairs having similar spectral types. A histogram (Figure 4) of the distribution of spectral type intervals (the difference between subclasses) indicates that most intervals are less than or equal to three subclasses. It may be suspected, therefore, that the correlations discussed in the previous section are due to this fact and have nothing to do with rotational correlation as such.

Let $F(u)$ denote the distribution of observed rotational velocities of stars in the sample (each component considered as a single star) and let $P(|\Delta u|)$ be the probability

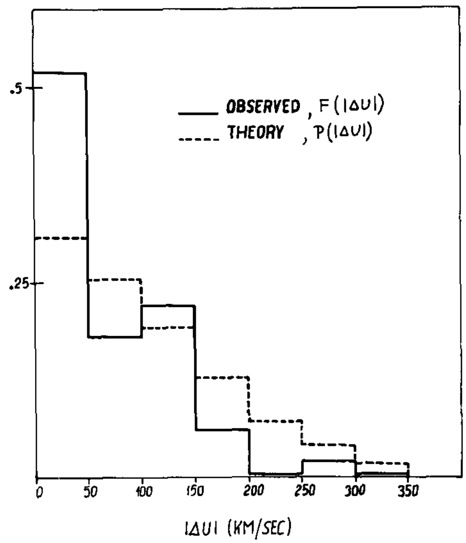

Fig. 5. Comparison of the distribution of rotational velocity differences in real pairs to the theoretical distribution (autocorrelation function).

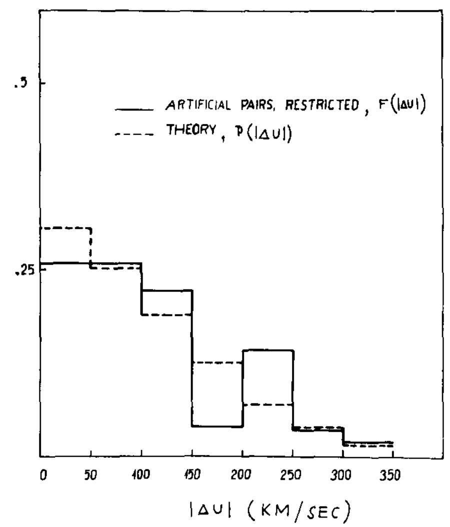

Fig. 6. Comparison of the distribution of rotational velocity differences in restricted artificial pairs to the theoretical distribution.

that two stars chosen at random have an observed rotational velocity difference $\Delta u = u_1 - u_2$. This probability is given by the autocorrelation function of $F(u)$.

Figure 5 shows $P(|\Delta u|)$ as well as the distribution of the differences $|\Delta u|$ in the real pairs. It is evident that the latter distribution is more sharply peaked towards small values of Δu, compared to the theoretical distribution $P(|\Delta u|)$.

From the sample of binaries, new pairs were matched at random but with the restriction that their difference in spectral subclasses is $\leqslant 2$ (i.e., a restriction more severe than on the real pairs). The distribution of the differences $|\Delta u|$ in these pairs is compared to the theoretical distribution $P(|\Delta u|)$ in Figure 6. As can be seen, there is an appreciably closer correspondence between these last two distributions than those in Figure 5.

Following Slettebak (1963), the mean velocities of rotation of various subclasses of

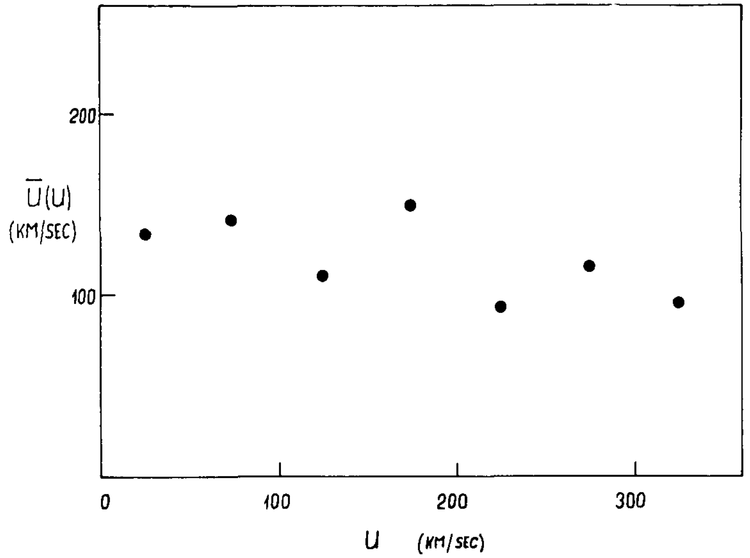

Fig. 7. Regression of the mean rotational velocities of components in artificial restricted pairs (*ordinate*) as a function of the rotational velocity interval of the other component (*symmetrized distribution*).

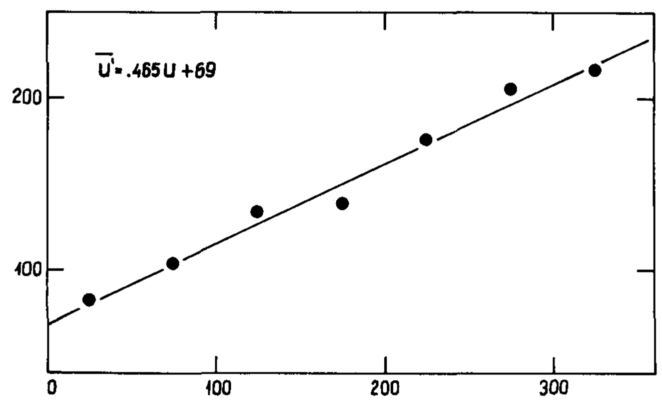

Fig. 8. Regression of the mean rotational velocities of components of true binaries (*ordinate*) as a function of the rotational velocity interval of the other component (*symmetrized distribution*).

the sample were derived (Table IV). From the small range in $\bar{u}$ (the mean velocity), it seems clear that a regression of one component of a binary on the other, i.e., the conditional mean velocities, cannot vary over a large range either, irrespective of the fact that their spectral types are similar. In order to check this, an analysis similar to the one carried out on the true binaries (see Subsection 5A) was performed on the artificial pairs restricted as to their difference in spectral type.

In order to obtain a more homogeneous sample, the distribution $F(u_1, u_2)$ was symmetrized, so that it is immaterial which star is the primary and which the secondary.

TABLE IV

Average velocity for various spectral types in the sample of visual binaries

Spectral type	No. stars	$\bar{u}$
B0–B3	15	158
B5–B7	8	148
B8–A2	47	158
A3–A7	14	146

For comparison purposes, the same symmetrization was applied to the sample of artificial binaries.

The results are:

(1) The correlation coefficient is $r=-0.11$ compared to $r=0.45$ for the real distribution.

(2) The regression $\bar{u}(u)$ of the artificial sample and that of the real sample are represented in Figures 7 and 8, respectively. The difference is evident.

In view of the results obtained in this section, we conclude that the regressions shown in Figures 2, 3, and 8 cannot be only the result of the fact that the visual binaries in our sample are heavily biased in favor of pairs having similar spectral types.

6. Concluding Remarks

If fission (Jeans, 1929; Roxburgh, 1966) is to account for the formation of binary systems, then one would in fact expect a high degree of rotational correlation to be present in binaries. In that sense our findings lend support to this theory.

However, it is impossible at this stage of the development of the theory of the origin of binaries to use our findings as a test case to support or disprove the capture and separate nuclei theories (see, e.g., Batten, 1967 for a summary of these theories).

It has been claimed that in some systems synchronization between axial and orbital rotations is present (Shajn and Struve, 1929), but it is difficult to see how this can be ascertained without knowing *a priori* that spin axes of members of binaries are in fact perpendicular to the orbital plane. Progress in this direction could be achieved in the following way:

Let $f_s(v_{sb})$ be the distribution of *true* rotational velocities of members of spectroscopic binaries, regarded as single stars, and let $F_{eb}(u)$ be the *observed* rotational velocities of members of *eclipsing binaries.*

If it is true that the spin axes are parallel to the orbital angular momenta, we should find that

$$F_{eb}(u) = f_s(v_{sb}). \tag{18}$$

As usual, we emphasize the need for more observational data. This would permit a more extended analysis to be carried out.

As a final remark, it should be pointed out that the idea of correlation in binary systems can be generalized to any property p measured in a quantitative way by testing whether or not

$$F(p_1, p_2) = f_1(p_1) \cdot f_2(p_2) \tag{19}$$

where $F(p_1, p_2)$ is the bivariate distribution function and $f_1(p_1)$ and $f_2(p_2)$ are the respective distributions for the components regarded as single stars.

References

Ambartsumian, V. A.: 1937, *Russian Astron. J.* **14**, 207.
Batten, A. H.: 1967, *Ann. Rev. Astron. and Astrophys.* **5**, 25.
Bečvář, A.: 1964, *Atlas Coeli Katalog II*, Československé Akadamie Věd, Prague.
Chandrasekhar, S.: 1944, *Astrophys. J.* **99**, 54.
Huang, S.-S. and Wade, Clarence Jr.: 1966, *Astrophys. J.* **143**, 146.
Jeans, J. H.: 1929, *Astronomy and Cosmogony*, Cambridge University Press, Cambridge.
Roxburgh, I. W.: 1966, *Astrophys. J.* **143**, 111.
Shajn, G. and Struve, O.: 1929, *Monthly Notices Roy. Astron. Soc.* **89**, 222.
Slettebak, A.: 1963, *Astrophys. J.* **138**, 118.
Struve, O.: 1950, *Stellar Evolution*, Princeton University Press, Princeton, N.J.
Trumpler, R. J. and Weaver, H. F.: 1953, *Statistical Astronomy*, University of California Press, Berkeley, Calif. U.S.A.