and Jupiter. Short-period terms depending on the two mean anomalies are eliminated from the disturbing function by von Zeipel's transformation.

Hori is now engaged in the computation of the secular variations of periodic comets by avoiding the usual expansion in powers of eccentricities, inclinations and the ratio of the major-axes.

It should be mentioned that Jeffreys (**26**) pointed out the similarity of von Zeipel's theory and Brown's theory for eliminating all short-period terms simultaneously from the disturbing function.

Miachin (**27**) has presented a general technique for estimating an error in numerical methods of integrating the differential equations of celestial mechanics, with specific reference to the methods of Cowell and of Runge. Kulikov (**28**) has worked out a procedure of integrating the equations of motion in celestial mechanics by using electronic computers and Cowell's quadrature method with automatic pitch selection.

BIBLIOGRAPHY

1. Garfalo, A. M. *Astr. J.*, **65**, 117, 1960.
2. Pines, S. *Astr. J.*, **66**, 5, 1961.
3. Cohen, C. J., Hubbard, E. C. *Astr. J.*, **67**, 10, 1962.
4. Herget, P. *Astr. J.*, **67**, 16, 1962.
5. Kaula, W. M. *Geophys. J. RAS.*, **5**, 104, 1961.
6. Musen, P. *J. Geophys. Res.*, **68**, 1439, 1963.
7. Popovič, B. *Radovi Nauč. Društva B. H.*, XIV, Sarajevo, 1961.
,, *Bilten Druš. Mat. Fiz. Makedonije*, X, Skopje, 1961.
,, *Bulletin Soc. Math.-Phys. de Serbie*, XIV, Beograd, 169, 1962.
,, *Ann. Acad. Sci. Fennicae, Ser. A*, **3**, no. 61, 209, 1961.
8. Danby, J. M. A. *Astr. J.*, **67**, 287, 1962.
9. Musen, P., Carpenter, L. *J. Geophys. Res.*, **68**, 2727, 1963.
10. Musen, P. *J. astronaut. Sci.*, **8**, 48, 1961.
11. Merton, G. *Mem. R. astr. Soc.*, **68**, no. 4, 1962.
12. ,, *M.N. RAS*, **109**, 421, 1949.
13. Sehnal, L. *Bull. astr. Inst. Csl.*, **7**, 125, 1956.
14. ,, ,, ,, , **10**, 67, 1959.
15. Musen, P. *J. Geophys. Res.*, **66**, 1659 and 2797, 1961.
,, *IUTAM Symposium on Dynamics of Satellites*, Paris, 1962, ed. by M. Roy. Springer, Berlin, 1963.
Musen, P. *Proc. First Intern. Symposium on the Use of Artificial Satellites for Geodesy*, Washington, D.C., 1962, p. 108, ed. by G. Veis. North-Holland Pub. Co., Amsterdam, 1963.
16. ,, *Rev. of Geophysics*, **1**, 85, 1963.
17. ,, *J. Geophys. Res.*, **66**, 659, 1961; **66**, 2797, 1961.
18. Clemence, G. M. *Astr. Pap. Amer. Eph. Naut. Almanac*, **16**, no. 2, 261, 1961.
19. ,, *Astr. J.*, **65**, 21, 1960.
20. Roemer, E. *Astr. J.*, **66**, 368, 1961.
21. Brandt, J. C. *Astrophys. J.*, **133**, 1091, 1961.
22. Musen, P. *J. Geophys. Res.*, **68**, 1439, 1963.
23. ,, *J. Geophys. Res.*, **64**, 2271, 1959.
24. Bailie, H., Bryant, R. *Astr. J.*, **65**, 451, 1960.
25. Kozai, Y. *Astr. J.*, **67**, 591, 1962.
26. Jeffreys, H. *M.N. RAS*, **122**, 335, 1961.
27. Miachin, V. F. *Bjull. Inst. teor. Astr.*, **8**, no. 8, (101), 537, 1962.
28. Kulikov, D. K. *Bjull. Inst. teor. Astr.*, **7**, no. 10, (93), 770, 1960.

CHARACTERISTIC ASTEROIDS

The long-period inequalities in the Keplerian elements of the characteristic asteroid Hilda

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due to the perturbation of Jupiter and Saturn for 273 years from 15 January 1875, are computed and the long-range behaviour is discussed by Akiyama (1), and those of the asteroid Thule due to the perturbation of Jupiter moving on a fixed ellipse for 400 years by Takenouchi. (2).

According to a letter from Schubart (3) he is working on the motion of asteroids with mean motions nearly commensurable with that of Jupiter. Following Poincaré's idea on the motion of the Hecuba group of asteroids the influence of the secular and the critical terms of the disturbing function is studied. These terms are obtained by numerically averaging the disturbing function of the restricted three-body problem in two or three dimensions with the aid of an IBM 7094 computer.

Message wrote me that he was engaged in the study of nearly commensurable mean motions in the restricted problem of three bodies, including some numerical investigation for the case $n/n' = 2/1$, with consideration of the effect of the long-period librations on the distribution of asteroids over mean motion near that case.

Wilkens wrote me that he was still working on the same problem.

Hagihara (4), by noticing that the mean motions of the existing asteroids belonging to the Hilda and the Thule groups are almost in commensurable ratios with that of Jupiter, at the same time as the several pairs of the satellites in the Saturnian system, while there are gaps corresponding to the ratios 2/1, 3/1, 5/2, 7/3 with large numbers of asteroids nearby, as well as gaps in the Saturnian ring, tentatively proposed a supposition that the large amplitudes of librations in the motion of the Hecuba group asteroids among others may be due to the disturbing effect of the neighbouring asteroids in the same group occasionally passing close by, as Brouwer mentioned in his paper on asteroidal families.

Rabe (5) announced his result of computation on the motion of the asteroid Griqua which has passed across the Hecuba gap quickly. Wilkens's computation has shown that the motion of an ideal asteroid in the exact commensurability-point is in libration around the exact commensurability of the mean motions. Similar circumstance may be thought of as to the Hilda and Thule group asteroids. Yet there exist asteroids in almost exact commensurability-points corresponding to the ratios 3/2 and 4/3.

Moser (6) proved the instability of the motion of characteristic asteroids for $p - q = 1, 2, 3, 4$, where $n/n' = p/q$, on purely gravitational ground.

Klose (7), by referring to the Jacobi integral, saw that the perturbation of the major axis is larger when the eccentricity increases than when it decreases. The distribution of major axes shows maxima at the commensurabilities of the types 2/1, 5/2 and 3/1. He plotted the distribution of the Jacobi constant and saw that the maxima in the distribution of major axes occurred by the drifting to smaller values of the major axes from the commensurability-points, accompanied by larger values of the eccentricity.

Brouwer's recent work (8) on the explanation of the gaps in the mean motions of asteroids is worth admiring. By von Zeipel's transformations he obtained a set of canonical equations in new variables that differed from the original by the sums of periodic terms with at least the first power of μ, the disturbing mass, as a factor. The new Hamiltonian is expanded in powers of μ. In the transformation process, divisors of the form $(j_1 + j_2 + j_3)n - j_1$ appear, where $n = 1/L^3$ and j_1, j_2, j_3, j_4 are integers. He distinguished two cases. (i) The ordinary case in which divisors of this form and small enough to endanger the convergence of the process do not appear. Two first integrals are obtained and the problem is reduced to that of two degrees of freedom. (ii) The commensurability case in which n is close to a commensurability $(p + q)/p$, where p and q are relatively prime positive integers. Then terms with arguments satisfying $(j_1 + j_2 + j_3)(p + q) = j_1 p$ will appear, which prevent the elimination of each term by a further transformation. The two integrals coalesce into a single integral in this case, but the problem is of three degrees of freedom. One of the integrals in the ordinary case is a substitute for the Jacobi integral.

Now suppose that the distribution of asteroids considered as a function of the Jacobi constant has no singularity, in the same way as Klose's. Brouwer found that the distribution function for L^*, which is the transform of L, is flat in the ordinary case, but becomes v-shaped in the commensurability case.

In the neighbourhood of a commensurability of a lower order, that is, in the ratio of two small integers, there exist an infinite number of commensurabilities in the ratios of very large integers, each with the v-shaped distribution functions. The higher order commensurabilities have adequate room for the v-shaped distribution in the vicinity of the 2/1 commensurability and also for commensurabilities such as 3/1, 5/2, 7/3, all with larger mean motions than 2/1. Brouwer says that for the 3/2 and even more for the 4/3 commensurability little room is available free from interference by other commensurabilities. This would, according to Brouwer, indicate the reason why the asteroids in this part of the belt seem to seek the commensurability region rather than avoid them. Brouwer thinks this conjecture is confirmed by the fact that the asteroids in the region between 440″ and 575″ mean motion show a distribution that is closely correlated with the distribution of the commensurabilities of comparatively lower orders. It can be understood that in the latter types of commensurabilities the v-shaped distribution may become almost flat due to the superposition or overlapping of the v-shaped distribution corresponding to the various commensurabilities of higher orders nearby. But the question is left unanswered why the asteroids in this part of the belt seem to seek the commensurability region. There is also the question of the relative density in the distribution of the commensurabilities of higher orders in the neighbourhood of the two types of the lower commensurabilities.

BIBLIOGRAPHY

1. Akiyama, K. *Publ. astr. Soc. Japan*, **14**, 164, 1962.
2. Takenouchi, T. *Ann. Tokyo astr. Obs.*, **7**, 191, 1962.
3. Schubart, J. *Smithsonian astrophys. Obs.*, *Special Report*, no. 149, 1964.
4. Hagihara, Y. *Smithson. Contr. Astrophys.*, **5**, no. 6, 1961.
5. Rabe, E. *Astr. Nachr.*, **271**, 92, 1941.
   ,, *Astr. J.*, **64**, 53, 1959.
6. Moser, J. *Astr. J.*, **63**, 439, 1958.
7. Klose, A. *Vierteljahrsschrift astr. Gesell.*, **63**, 333, 1928.
8. Brouwer, D. *Astr. J.*, **68**, 152, 1963.

TROJAN ASTEROIDS

Rabe (1) carried out the actual determination on electric computers of periodic libration orbits about the equilateral triangular equilibrium points of the restricted problem and their stabilities. He at first describes the general method developed for finding such periodic orbits and then deals with orbits of various dimensions including one of the horse-shoe shaped in the natural Trojan case (1) and, together with Schanzle, in the case of the Earth-Moon restricted problem (3). Rabe (2) obtained the Fourier series representation of all orbits for asteroids of the Trojan group by numerical harmonic analysis with an electronic computer. The convergence of the Fourier expansions is very satisfactory, according to Rabe, up to amplitudes of the order of those of the actual Trojan asteroids with the largest libration amplitudes. All orbits computed appear to be stable, even the horse-shoe shaped periodic orbit enclosing both the equilateral triangular equilibrium points L_4 and L_5 and the collinear equilibrium point L_3 opposite to Jupiter from the Sun. This latter class of orbit, anticipated by Brown, is of particular interest, because with their increasing amplitude but decreasing periods these orbits link the equilibrium

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