problems but are an essential tool in their analysis. There is a vast wealth of information in this chapter, which is possibly the most valuable of the whole book.

This volume can be highly recommended for inclusion in every mathematics library. It is well produced, with exercises and references at the end of each chapter.

IAN ANDERSON

MULDOWNEY, P. *A general theory of integration in function spaces, including Wiener and Feynman integrals* (Pitman Research Notes in Mathematics Series **153**, Longman Scientific and Technical, Harlow 1987), pp. 115, 0 582 99465 9, paper, £12.

In this monograph the theory of the Henstock integral is expanded, and applied to the path integrals named after Wiener and Feynman.

The Henstock integral is a generalization of the Riemann integral combining simplicity with considerable power. Recall that a function $f(x)$ defined for $0 \leqq x \leqq 1$ is said to be Riemann-integrable with integral c if for all sufficiently fine partitions

$$0 = a_0 < a_1 < \cdots < a_n = 1$$

and for all attached points $a_{i-1} \leqq x_i \leqq a_i$ the finite sum $\sum f(x_i)(a_i - a_{i-1})$ approximates c. Here the fineness of the partition is measured by the mesh $\delta = \sup\{a_i - a_{i-1}\}$. Henstock's generalization consists in allowing the mesh to depend on the point x_i, so that in the course of the $\varepsilon - \delta$ argument formalizing the above the fineness condition becomes

$$a_i - a_{i-1} \leqq \delta(x_i)$$

for some positive function $\delta(x)$.

Henstock's procedure applies as well to integrals of infinite range, and agrees with the Lebesgue integral when that applies. Moreover it applies to some integrands of varying sign which are *not* absolutely integrable. This is indeed its major strength, although for this reason it clearly cannot be related to an underlying measure theory and so its various convergence theorems require more careful statement than those of the Lebesgue integral.

To fulfil the intention of the monograph it is necessary to apply the Henstock procedure to domains which are product spaces (indeed, of uncountably many factors). It will come as no surprise to measure-theorists that the simplicity of the original definition becomes less apparent; product spaces are the source of several technical difficulties for more orthodox integration theories also. The technicalities do however appear to differ in an interesting way. For example if the space of continuous functions is viewed as a subset of $\mathbb{R}^{[0,1]}$ then its indicator function is immediately Wiener-integrable if the Henstock procedure is employed.

What is lost and what is gained by using the Henstock integral?

Considerations of simplicity apart, we know *a priori* little or nothing can be gained in the theory of integration of nonnegative functions. Recall the summary of Solovay (1970): "the existence of a non-Lebesgue measurable set cannot be proved in Zermelo–Fraenkel set theory if use of the axiom of choice is disallowed." This confirms the empirical experience of analysts and probabilists, that Lebesgue integration theory is sufficient for the demands of absolute integration. The Henstock integral encompasses the Lebesgue theory for nonnegative functions; whether by strict inclusion is not clear from the monograph but any gain in this direction must clearly be of some sophistication in the style of mathematical logic. For practitioners, the simplicity of Henstock's definition must be matched against the enormous expressive power of the measure theory underlying Lebesgue's integral.

In the case of non-absolute integration Henstock's procedure provides for example an unambiguous value for Fresnel's integral $\int_0^\infty \exp(iy^2)\,dy$, which of course does not exist in Lebesgue's sense. The attraction of Henstock's theory is that it provides an automatic procedure for evaluating such integrals, together with a reasonable quantity of limit theorems and in particular a proper link to the differential calculus. I do however have a worry here; it is not clear in the abstract setting precisely how to link these nonabsolute integrals to topological and metric properties of the underlying space.

This worry can be stated clearly in the context of the Wiener integral. The metric and topology of continuous path-space are closely and explicitly linked to the usual measure-theoretic development. The Henstock integral as expounded in the monograph applies directly, avoiding metric and topological niceties, to produce a non-absolute integral generalizing the conventional Wiener integral. But unambiguous *interpretation* of this extension in metric and topological terms appears still to be lacking. And much of the richness of the orthodox theory lies in its interpretations.

Incidentally, readers should note the extensive and important generalization of the Wiener integral to the stochastic or Itô integral (see Rogers and Williams, 1987, for a recent exposition).

The Feynman integral is to the Wiener integral as $\int_0^\infty \exp(ix^2)\,dx$ is to $\int_0^\infty \exp(-x^2)\,dx$, but with even more difficulty imposed by the infinite-dimensional character of path-space. In this setting conditional or non-absolute integration has an essential role to play. The monograph shows how Henstock's procedure can be applied, and relates the ensuing definition to definitions and results due to Nelson, Cameron, and Truman. (It is not however clear to me whether the Henstock version is as general as that proposed in Elworthy and Truman, 1984.)

In conclusion, this monograph provides a useful exposition in book form of Henstock integration theory as applied to path integrals, and we owe the author thanks for this. However I believe there is still wanting an exposition which interprets Henstock's constructions in terms which can be directly related to constructions in measure theory, topology, and geometry.

REFERENCES

K. D. Elworthy and A. Truman, Feynman maps, Cameron-Martin formulae and anharmonic oscillators, *Ann. Inst. H. Poincaré Phys. Théor.* **41** (1984), 115–142.

L. C. G. Rogers and D. Williams, *Diffusions, Markov processes and martingales*, Vol. 2 (Wiley, Chichester 1987).

R. Solovay, A model of set-theory in which every set of reals is Lebesgue measurable, *Ann. of Math.* **92** (1970), 1–56.

W. S. KENDALL

Blackadar, B. *K-theory for operator algebras* (Mathematical Sciences Research Institute Publications Vol. 5, Springer-Verlag, New York–Berlin, 1986), pp. 337, 3 540 96391 X, DM 78.

The introduction of K-theory in operator algebras in the seventies has revolutionized C*-algebra theory, and led to several major advances since then. Though there have been several conferences on K-theory in C*-algebras, there was no unified account of the subject until this book appeared. Blackadar has set himself the task of giving a comprehensive account of K-theory in operator algebras with the exception of the applications of the theory. He has succeeded in taking the subject from its beginnings to its most recent advances in Kasparov's KK-theory. The tremendous range of mathematics covered in this book means that some of the proofs are given in the detail of a monograph rather than a textbook or lecture notes.

The book assumes the basic theory of Banach algebras and C*-algebras. A familiarity with ideas from topology is helpful for motivation and for several of the analogies implicitly drawn. The book starts with brief "overviews" of topological and operator K-theory to motivate the subsequent mathematics and detailed definitions. The main discussion begins with the definition of $K_0(A)$ of a C*-algebra A as a group with order. The definition is in terms of algebraic equivalence classes of idempotents in $M_\infty(A)$, where $M_\infty(A)$ is the inductive limit of the $n \times n$ matrix algebras $M_n(A)$ over A. There is of course a little twist that is required in the definition if A is not initial. However, here there is probably too little detail for the novice reader, a point which will reduce the usefulness of the book to some potential readers. For example, the crucial Whitehead matrix calculations that are so clear in J. L. Taylor's account of K-theory ("Banach algebras and topology" in *Algebras in analysis*, ed. J. H. Williamson, Academic Press 1975), are